Making the “Green Machine”

To make the cards from scratch, the students need to reverse this thinking to discover the relationship between the secret number and the cards that received a “yes” answer. The special numbers identified on the five cards in the lesson are 1, 2, 4, 8, and 16. Each number is placed on a large individual card with magnetic tape, and the cards are posted on the front chalkboard.

1	**2**	**4**	**8**	**16**

Students then fold three sheets of 8 1/2-by-11-inch green construction paper in half to create six 8½” × 5½” cards (although they will only need five of them). Arrange five cards in a row in front of the students from left to right with the numbers 1, 2, 4, 8, and 16 placed, respectively, in the upper-left corners:

1	*2*	*4*	*8*	*16*

Using the numbers posted on large cards at the chalkboard, the students are asked to generate all numbers from 1 to 31 by finding combinations of the five special numbers that add up to each number. Students should place each number on the appropriate card(s). For instance, the number 1 is generated by adding just the 1-card; the number 2 is generated by adding just the 2-card; the number 3 is generated by adding the 1-card and the 2-card, because 1 + 2 = 3; and so on. (The numbers 1, 2, and 3 have been placed on the appropriate cards below.) Thus, a person who has selected 3 as the secret number will say “yes” to the 1-card and to the 2-card (so the magician mentally adds 2 + 1, getting 3), but the person will say “no” to the 4-card, the 8-card, and the 16-card. Therefore, the person’s secret number is 3.

1 1 3	*2* 2 3	*4*	*8*	*16*

The cards are filled with numbers in this manner by having the students offer ways to make the combinations, which also reinforces mental addition with these five numbers. When completed, the students put the cards in order — the 1 card on top and the 16 card on bottom — and staple them

together. To assess their understanding of the activity and check for success, the students try to guess their neighbor's secret number.

The students will want to know why this procedure works. The 1, 2, 4, 8, and 16 that they identified as "special numbers" should be examined for other relationships. The special numbers could more appropriately be called, "the place values of the binary number system," or, "the exponential values of 2." The students were adding all the combinations of the binary place-value numbers to make the numbers from 1 to 31.

For young students, use four cards for numbers up to 15 to make the investigation of patterns easier. With fewer possibilities, students are more likely to discover the patterns on their own. Once they begin to recognize the binary patterns, they are more likely to seek and find binary patterns in more complicated situations.