Dirt Bike Dilemma

NAME ____________________________

The Annual Springfield Dirt Bike Competition is coming up, and participants are looking for bikes! Of course, they turn to Apu, who has the best bikes in town.

Apu has 18 wheels, 15 seats, and 14 exhaust pipes in his supply room. He can use these parts to assemble two different types of bikes: The Rider, or The Rover.

> The Rider has 2 wheels, 1 seat, and 2 exhaust pipes. It is designed to glide around curves effortlessly.
>
> The Rover has 3 wheels, 3 seats, and 1 exhaust pipe. It is designed to carry multiple passengers over the roughest terrain.

Apu needs to decide how many of each bike he should assemble to maximize his profit. Because of the popularity of the Dirt Bike Competition, he knows that no matter how many bikes he assembles, he will be able to sell all of them. Apu requests your assistance in making this decision.

Every member of your team should have the following items:

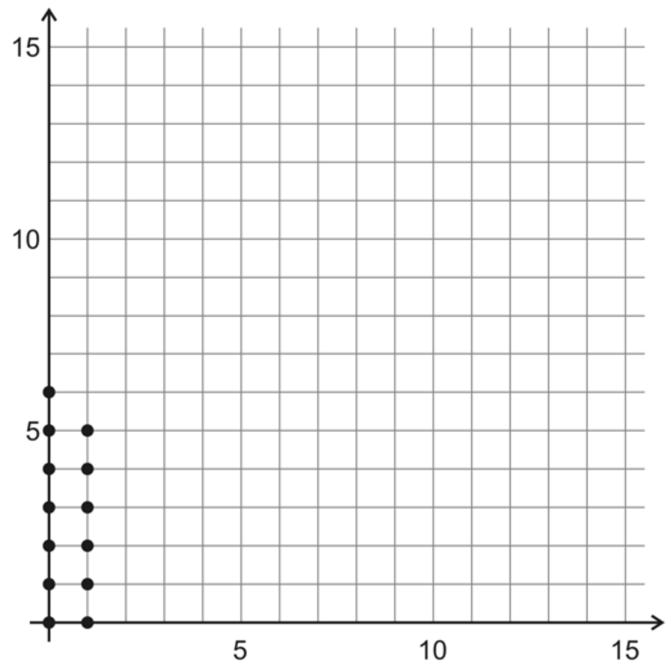
- Graphing Calculator
- Dirt Bike Dilemma Activity Sheet
- Three (3) Colored Pencils
- Set of Cards

In addition, each member of your team should get some cards:

- ❑ One member of your team should get 18 Wheel Cards. This person should complete Question 1.
- ❑ Another member of your team should get 14 Exhaust Pipe Cards. This person should complete Question 2.
- ❑ The last member of your team should get 15 Seat Cards. This person should complete Question 3.

1. Given 18 wheels, list all possible combinations of Riders and Rovers that can be assembled. Remember that each Rider needs two wheels and each Rover needs three wheels. Using only the wheel cards, complete the table. Plot the data on the grid below. (The possible combinations from the first two rows have been plotted for you.)

NUMBER OF RIDERS	POSSIBLE NUMBER OF ROVERS
0	0, 1, 2, 3, 4, 5, 6
1	0, 1, 2, 3, 4, 5
2	
3	
4	
5	
6	
7	
8	
9	



a. What do you notice about the graph?

____

____

____

b. On your graph, draw a line that borders all the points. This line should pass through *some* of the points that represent the maximum number of Rovers.

What inequality could be used to represent this relationship? ________________________
This inequality is called a *restriction* or a *constraint*.

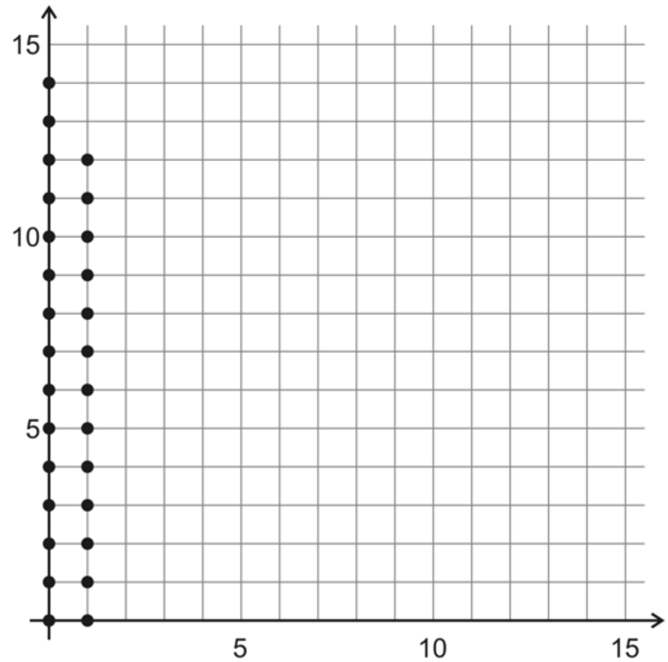
c. How can you arrive at this inequality without the use of the table and graph?

____

____

2. Given 14 exhaust pipes, list all possible combinations of Riders and Rovers that can be assembled. Remember that each Rider needs two exhaust pipes and each Rover needs one exhaust pipe. Using only the exhaust pipe cards, complete the table. Plot the data on the grid below. (The possible combinations from the first two rows have been plotted for you.)

NUMBER OF RIDERS	POSSIBLE NUMBER OF ROVERS
0	0, 1, 2, …, 14
1	0, 1, 2, …, 12
2	
3	
4	
5	
6	
7	



a. What do you notice about the graph?

____

____

____

b. On your graph, draw a line that borders all the points. This line should pass through *all* of the points that represent the maximum number of Rovers.

What inequality could be used to represent this relationship? ___________________________
This inequality is called a *restriction* or a *constraint*.

c. How can you arrive at this inequality without the use of the table and graph?

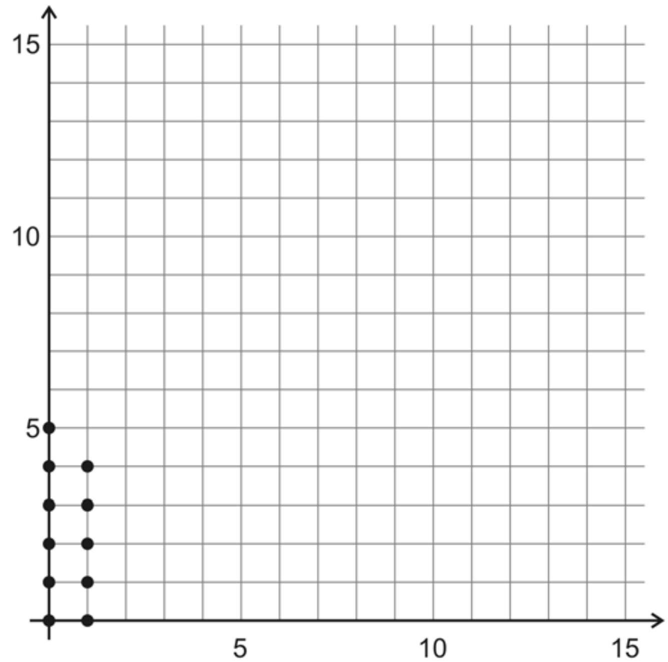
____

____

____

3. Given 15 seats, list all possible combinations of Riders and Rovers that can be assembled. Remember that each Rider needs one seat and each Rover needs three seats. Using only the seat cards, complete the table. Plot the data on the grid below. (The possible combinations from the first two rows have been plotted for you.)

NUMBER OF RIDERS	POSSIBLE NUMBER OF ROVERS
0	0, 1, 2, 3, 4, 5
1	0, 1, 2, 3, 4
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	



a. What do you notice about the graph?

____

____

____

b. On your graph, draw a line that encloses all the points. This line should pass through some of the points that represent the maximum number of Rovers.

What inequality would represent this relationship? ____________________
This inequality is called a *restriction* or a *constraint*.

c. How can you arrive at this inequality without the use of the table and graph?

____

____

____

4. Discuss your answers with your team members. Explain how you arrived at your responses. Based on your discussion, complete Questions 1 through 3.

If all of the ordered pairs (Rider, Rover) that are feasible options are identified in the three graphs above, explain why each statement below is true.

a. All ordered pairs have integer coordinates.

____

____

____

b. When graphed in the coordinate plane, all ordered pairs will be located in either the first quadrant or on the positive *x*-axis or *y*-axis.

____

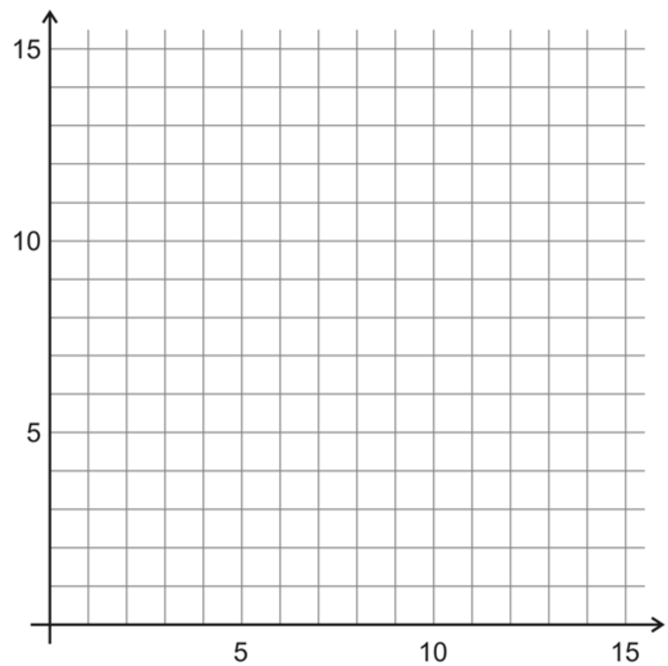
____

____

5. Below, list the three inequalities from Questions 1b, 2b, and 3b. Since all feasible ordered pairs (Rider, Rover) must be located either in the first quadrant or on one of the positive axes, what TWO additional inequalities should also be added to this list? Add them below.

6. Put all of your cards together. As a team, using the cards and the information from Questions 1-3, determine all possible combinations of Riders and Rovers that can be assembled with 18 wheels, 15 seats, and 14 exhaust pipes. Remember that each Rider needs 2 wheels, 1 seat, and 2 exhaust pipes, and each Rover needs 3 wheels, 3 seats and 1 exhaust pipe. Complete the table below, and plot your data on the grid.

NUMBER OF RIDERS	POSSIBLE NUMBER OF ROVERS
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	



7. Carefully graph all five inequalities from Questions 5 on the grid in Question 6. What do you notice?

____

____

____

____

The region bounded by these inequalities is called the *feasible region*. The feasible region is the region that satisfies **all** of the constraints.

8. Suppose Apu makes a profit of $15 for each Rider and $30 for each Rover. Select two points from the feasible region to determine the total profit that Apu would receive. Show how you arrived at your answers.

a. First point in the feasible region: (_____ , _____)

b. Second point in the feasible region: (_____ , _____)

9. If Apu makes a profit of $15 on each Rider and $30 on each Rover, write an expression to represent the total profit he receives. Let *x* represent the number of Riders he sells, and let *y* represent the number of Rover he sells.

Total Profit = ____

This function is known as an *objective function*. The objective function is the function that you are trying to maximize or minimize. (In this case, the objective is to maximize Apu's profit.)

10. Apu makes a profit of $15 for each Rider and $30 for each Rover.

a. Find three ordered pairs in which the total profit earned would be $90, $120, or $180. (The points you select do not have to be in the feasible region.)

PROFIT	ORDERED PAIRS		
$90	(___ , ___)	(___ , ___)	(___ , ___)
$120	(___ , ___)	(___ , ___)	(___ , ___)
$180	(___ , ___)	(___ , ___)	(___ , ___)

b. On the grid below, plot each set of points (those for a total profit of $90, those for a total profit of $120, and those for a total profit of $180) in a different color. Each set of three points should form a straight line. Why does this make sense?

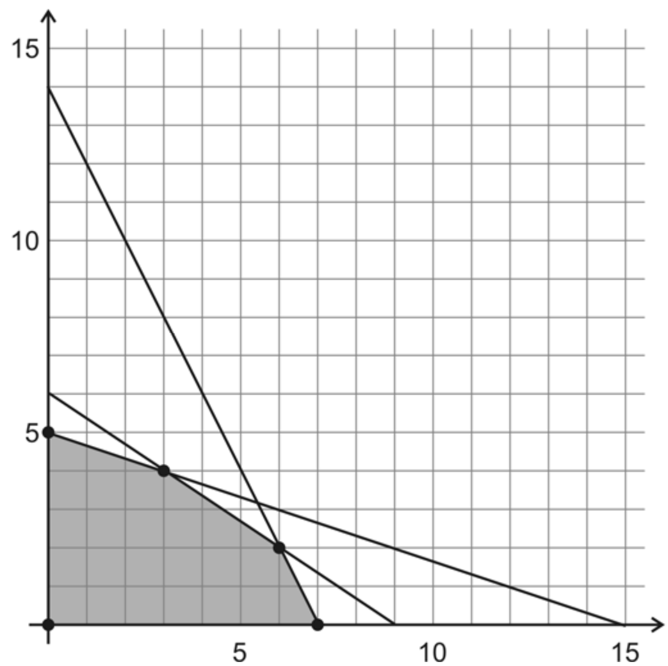
____

____

Draw a line through each set of points. What do you notice about these lines? Why does this make sense?

____

____



c. Does one of these values — $90, $120, or $180 — represent the MAXIMUM total profit that Apu can earn if he receives a profit of $15 for each Rider and $30 for each Rover? Explain your reasoning.

____

____

____

11. Using the TI-83+ or TI-84+ Graphing Calculator, follow the steps outlined below.

a. Press [PRGM]. Use the down cursor key to highlight **DRTBK**. Press [ENTER]. Press STAT. Press [ENTER].

b. Use the up cursor key to highlight **L1**. Press [2nd] [DEL]. Press [2nd] [STAT]. Use the down cursor key to highlight ***RIDER***. Press [ENTER] twice. This column represents the number of Riders sold.

c. Use the right and up cursor key to highlight **L1**. Press [2nd] [DEL]. Press [2nd] [STAT]. Use the down cursor key to highlight ***ROVER***. Press [ENTER] twice. This column represents the corresponding number of Rovers sold.

d. Use the right and up cursor key to highlight **L1**. Press [2nd] [DEL]. Press [2nd] [STAT]. Use the down cursor key to highlight ***TPRFT***. Press [ENTER] twice. This column represents the Total Profit received.

e. Use the right and up cursor key to highlight **L1**. Press [2nd] [DEL]. Press [2nd] [STAT]. Use the down cursor key to highlight ***PRFIT***. Press [ENTER] twice. The first number in this column represents the profit earned for each Rider sold and the second number represents the profit earned for each Rover sold.

f. Use the up cursor key to highlight the number below ***PRFIT***. Type in a value for the profit Apu receives for each Rider he assembles. Press [ENTER]. Type in a value for the profit Apu receives for each Rover he assembles. Press [ENTER].

In **Step f**, enter 15 as the profit for each Rider and 30 as the profit for each Rover. Move the cursor to the ***TPRFT*** column. Use the cursor key to find the *maximum total profit* (the largest number in this column).

Record this value in the appropriate space in the table below. Along with this value, record the corresponding values for Riders and Rovers. To change the profit earned on each Rider and Rover, move to the ***PRFIT*** column and repeat **Step f**. Complete the table belowchoosing your own values for the last several rows.

PROFIT ON EACH RIDER	PROFIT ON EACH ROVER	NUMBER OF RIDERS	NUMBER OF ROVERS	MAXIMUM TOTAL PROFIT
$15	$30			
$20	$20			
$10	$40			

Compare your results with those of your team members. Which combinations of (**Rider, Rover**) always appear?

____

Where are these points located on your graph in Question 6?

____

____

Given all the points in the feasible region, why do you think that just one (**Rider, Rover**) combination always yields the maximum profit?

____

____

____

12. Using your graphing calculator, follow the steps below.

Step 1:	Press [APPS]. Press the up cursor key. Use the down cursor key to highlight **TRANSFRM**. Press [ENTER] twice.
Step 2:	Press [WINDOW]. Press the up cursor key. Use the down cursor key to highlight step. Type in 5. Press [ENTER]. Press [GRAPH].
Step 3:	Use the up or down cursor key to move to **A**. Enter **15**. Press [ENTER]. **A** represents the profit earned for each Rider. Use the down cursor key to move to **B**. Enter **30**. **B** represents the profit earned for each Rover. Press [ENTER]. Use the down cursor to move to **C**. Type in **0**. Press [ENTER]. **C** represents the total profit earned.
Step 4:	Use the right cursor key to increase the value of **C**. Watch the line on your graph.

a. As the line moves, what is the last point in the feasible region through which the line passes?

(_____ , _____)

b. What is the value of **C** at this point? ____________

c. Repeat **Steps 3** and **4** for different values of **A** and **B**. As a team, come up with an explanation for why the corner points of the feasible region always yield the maximum (or minimum) profit.

____

____

____

13. Let's return to Apu's dilemma.

Apu sets his prices so that he will make a profit of $21 for every Rider he sells and $32 for every Rover he sells. Determine *algebraically* how many of each type he should assemble to receive the maximum profit. What is the maximum profit?

14. Look over your responses to Questions 4-12. Concentrate on the process needed to solve Apu's dilemma. Assume that you do not have access to a graphing calculator. As a team, discuss and list **five** major steps required to solve a problem of this type (which is known as a *linear programming problem*).

15. Use your steps from Question 14 to solve the problem below.

Lisa is making cookies to sell at the Annual Dirt Bike Competition. A dozen oatmeal cookies require 3 cups of flour and 2 eggs. A dozen sugar cookies require 4 cups of flour and 1 egg. She has 40 cups of flour and 20 eggs. She can make no more than 9 dozen oatmeal cookies and no more than 7 dozen sugar cookies, and she earns $3 for each dozen oatmeal cookies and $2 for each dozen sugar cookies. How many dozens of each type of cookie should she make to maximize her profit?