Place Value

NAME ____________________________

1. Notice that the only digits needed to write the integers in base-6 are 0, 1, 2, 3, 4, and 5.

(a) Using the repeated subtraction method, explain why a power of 6 is never subtracted more than five times.

(b) Using the repeated division method, explain why the remainder is never greater than 5.

2. Choose a four-digit base-6 number $abcd_6$. Of course, the digits *a*, *b*, *c*, and *d* should all be in the range 0-5.

(a) Find the decimal (base-10) equivalent to $abcd_6$.

(b) Use repeated subtraction to find the base-6 representation of the decimal number you found in part (a).

(c) Finally, use repeated division on the number obtained in part (a) to get the base-6 representation in a different way.

3. For each of the integers in the first column, use repeated division or repeated subtraction to find the base 2, base 4 and base 8 representations. Look for patterns as you complete the chart.

n	binary (base-2)	base-4	base-8
104	1101000	1220	150
105			
106			
107			
108			
109			
110			
111			
112			

4. Find positive integers a, b, c, and d, each less than 9, such that $a \cdot 9^3 + b \cdot 9^2 + c \cdot 9 + d = 2006$.

Can this be done in more than one way?

5. The Cantor (or factorial) representation of a number N is a string of digits $(d_n\, d_{n-1} \ldots d_1)_f$ satisfying $N = d_n \cdot n! + d_{n-1} \cdot (n-1)! + \ldots + d_1 \cdot 1!$ where $0 \leq d_i \leq i$. Thus, for example, $11 = 1 \cdot 3! + 2 \cdot 2! + 1 \cdot 1! = 121_f$. Note that the rightmost digit d_1 will either be 1 or 0 in all cases. How can you tell quickly which of the two values to use for d_1, given N?

Use repeated division to find the Cantor or factorial representation of each of the following numbers: 2005, 3005, 4005, and 5005. (Note: The subscript C is to indicate Cantor representation.)

2005 = _____ · 6! + _____ · 5! + _____ · 4! + _____ · 3! + _____ · 2! + _____ · 1!

= $\_\_\_\_\_\_\_\_\_\_\__C$

3005 = _____ · 6! + _____ · 5! + _____ · 4! + _____ · 3! + _____ · 2! + _____ · 1!

= __________ $_C$

4005 = _____ · 6! + _____ · 5! + _____ · 4! + _____ · 3! + _____ · 2! + _____ · 1!

= __________ $_C$

5005 = _____ · 6! + _____ · 5! + _____ · 4! + _____ · 3! + _____ · 2! + _____ · 1!

= __________ $_C$

6. Each column in the table below provides the representation of the numbers from 1 to 12 for certain system of enumeration. Of course, the entries in column A are decimal representations. Study the pattern, and determine the method of enumeration used in each column. Then, fill in the missing representations.

Pay special attention to columns B and C.

A	*B*	*C*	*D*	*E*	*F*
1	0	1	1	1	1
2	1	10	10	2	110
3	10	100	11	10	111
4	2	101	20	11	100
5	100	1000	21	12	101
6	11	1001	100	20	11010
7	1000	1010	101	21	11011
8	3	10000	110	22	11000
9	20	10001	111	100	11001
10	101	10010	120	101	11110
11	10000	10100	121	102	11111
12	12	10101	200	110	11100
13					
14					
15					

7. Count from 1 to 10 in Fibonacci. How do your results compare to one of the columns in the previous question. Now count from 1 to 20 in Fibonacci and describe any patterns you notice.

8. Find the sum of the Fibonacci numerals $101010_f + 100101_f + 101001_f$.

Challenge 1: In this problem, you will explore representation of integers in a negative base. For convenience, use $b = -6$. Use repeated division to find the base -6 representation of the base 10 number 2006. Division by -6 requires some extra care. The crucial observation is that the remainders must always be in the range 0 to 5, just as with base-6.

Next, see if repeated subtraction works as it did for positive bases.

Challenge 2: To find the base-2, base-4, or base-8 representations of a number, it is not necessary to convert to a base-10 representation.

a. Devise a method to find the base-8 and base-4 representations of a number based on its binary (i.e., base-2) representation without first converting to decimal.

b. Devise a method to find the binary representation given its quartic (i.e., base-4) representation without first converting to decimal.

c. Devise a method to find the binary representation given its octal (i.e., base 8) representation without first converting to decimal.

Challenge 3: Find positive integers a, b, c, and d, each less than 6, such that

$$\frac{a}{6} + \frac{b}{36} + \frac{c}{216} + \frac{d}{1296} = \frac{437}{1296}$$