

# Exact Ratios

NAME \_\_\_\_\_

When you recorded the lengths from the bridge to the nut and to various frets, did you find that all ratios were roughly the same? You should have seen that the ratio of consecutive segments was always close to .

In actuality, the exact ratio of consecutive segments is  $2^{-1/12}$ .

1. Use a calculator to approximate  $2^{-1/12}$ . Round the value to four decimal places.
2. How close were your ratios of consecutive string lengths to the value you found above? Discuss why there might be variations in your ratios, and explain why the ratios might not always be exactly  $2^{-1/12}$ .
3. A geometric sequence has the form  $a, ar, ar^2, ar^3, \dots, ar^n, \dots$ , where  $a$  is the first value of the sequence and  $r$  is a ratio. The  $n^{\text{th}}$  term of a geometric sequence is  $ar^n$ .

The values that you recorded in the chart approximately form a geometric sequence. Using the values from your chart, determine approximate values of  $a$  and  $r$ .

$$a \approx \underline{\hspace{2cm}}$$

$$r \approx \underline{\hspace{2cm}}$$

Explain how you determined the values of  $a$  and  $r$ .

4. What is the *exact* value of  $r$ ?

5. Using your approximate value of  $a$  and the exact value of  $r$ , determine the values of the middle column in the table below. In the right column, use your measurements. How do the values compare?

	CALCULATED LENGTH	MEASURED LENGTH
Nut to Bridge	$a =$	
2nd Fret to Bridge	$ar^2 =$	
4th Fret to Bridge	$ar^4 =$	
6th Fret to Bridge	$ar^6 =$	
8th Fret to Bridge	$ar^8 =$	
10th Fret to Bridge	$ar^{10} =$	
12th Fret to Bridge	$ar^{12} =$	

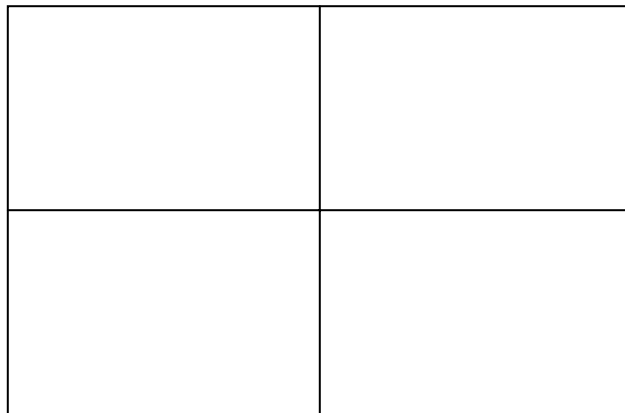
6. How does the value of  $a$ , the length from the nut to the bridge, compare to the value of  $ar^{12}$ , the length from the 12th fret to the bridge?
7. Play a string “open,” that is, without placing your finger on a fret. Then, play the same string by holding down the string just before the 12th fret. Compare the two notes. (If there is no musician in your group, find a musician in the class to help you compare the notes.)

8. The distance from the bridge to each fret can be described by an ordered pair of the form  $(n, d)$ . In this case,  $n$  is the number of the fret and  $d$  is the distance. For instance, if the distance from the bridge to the 8th fret is 15.7 cm, the ordered pair would be  $(8, 15.7)$ .

- Using the Stats Plot feature on a graphing calculator, plot the points corresponding to your measurements.
- Use the regression feature on your calculator to find an exponential equation (i.e., an equation of the form  $y = ar^x$ ) that goes through the points, and plot that curve. Record the equation below.

Exponential equation: \_\_\_\_\_

- Below, draw the scatter plot from 8a, the regression line from 8b, and the curve  $y = ar^x$  using the value of  $a$  from Question 3 and the value of  $r$  from Question 4. Record the max and min values of the view window.



$x$ -min: \_\_\_\_\_

$x$ -max: \_\_\_\_\_

$y$ -min: \_\_\_\_\_

$y$ -max: \_\_\_\_\_

9. Discuss the similarities and differences of the three plots from Question 8.

10. The three graphs from Question 8 should have the same  $y$ -intercept, approximately. What are the coordinates of the  $y$ -intercept?

$y$ -intercept: ( \_\_\_\_\_, \_\_\_\_\_ )

11. As it relates to your musical instrument, what does the  $y$ -intercept represent?