Iterating to Find the Square Root of Two ($\sqrt{2}$)

NAME _____

Pick some number that you know is greater than sqrt 2 as the first guess in determining this root. _____

1. Use the following formula for the "NEXT" guess:

$$NEXT = \frac{GUESS + \frac{2}{GUESS}}{2}$$

2. Your guess times the quotient of 2 divided by your guess equals 2:

$$GUESS \times \frac{2}{GUESS} = 2$$

If your guess is less than sqrt 2, then the quotient 2/GUESS will be greater than sqrt 2. The formula averages GUESS and 2/GUESS to yield NEXT, the next (better) guess.

What is your second guess?_____

3. To use this algorithm on a calculator, do the following:
   (a) Store your guess in A: On a TI-92, enter your guess and press STO A
   (b) On the command line of a TI-92, enter (A + (2/A))/2, and press STO A
   (c) Press ENTER several times. Write the sequence of values that you obtain.

   _____ _____ _____ _____

   _____ _____ _____ _____

   _____ _____ _____ _____

4. (d) Find sqrt 2 with your calculator. _____
5. (e) How accurate was the iterative algorithm you used?_____
6. _____

Discussion and Extension

1. Is there a way to represent this iterative process graphically in order to see that it really does find sqrt 2?_____.

Explain any ways you discovered._____
_____

2. Is there an algebraic argument for convincing someone that the iterative process used really does keep getting closer to sqrt 2? State your argument. _____
_____

3. How many iterations would be required to get one hundred-decimal place accuracy in the estimate of sqrt 2?
_____

4. If the process converges to sqrt 2, how "fast" does it converge? That is, how does the error change from iteration to iteration?

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