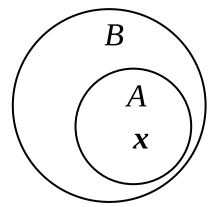
Reference Sheet

The following symbols are used on this sheet:

$\subset$ "is a subset of"	$\in$ "is an element of"	$\notin$ "is not an element of"	$\cap$ "intersects"

Direct Reasoning

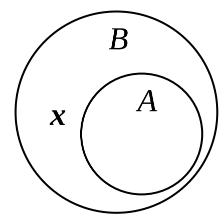


THE UNDERLYING IDEA

If $A \subset B$ and $x \in A$, then we can conclude that $x \in B$.

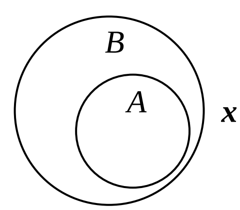
Premise:	All elements in *A* are also in *B*.	$p \rightarrow q$:	If an element is in *A*, then it is in *B*.
Premise:	*x* is an element in *A*.	p:	*x* is an element in *A*.
Conclusion:	*x* is an element in *B*.	$\therefore q$:	*x* is in *B*.

The argument is VALID because if *A* is in *B* and *x* is in *A*, then we can be 100 percent certain that *x* is in *B*.



But watch out! If $A \subset B$ and $x \in B$, we cannot be 100 percent certain that $x \in A$. The argument would be invalid. The diagram to the right shows that $A \subset B$ and $x \in B$, but $x \notin A$.

Indirect Reasoning

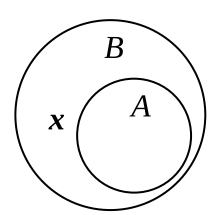


THE UNDERLYING IDEA

If $A \subset B$ and $x \notin B$, then we can conclude that $x \notin A$.

Premise:	All elements in *A* are also in *B*.	$p \rightarrow q$:	If an element is not in *B*, then it is not in *A*.
Premise:	*x* is not an element in *B*.	p:	*x* is not an element in *B*.
Conclusion:	*x* is not an element in *A*.	$\therefore q$:	*x* is not in *A*.

The argument is VALID because if *A* is in *B* and *x* is not in *B*, then we can be 100 percent certain that *x* is not in *A*.

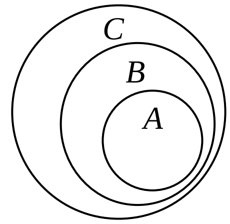


But watch out! If $A \subset B$ and $x \notin A$, we cannot be 100 percent certain that $x \notin B$. The argument would be invalid. The diagram to the right shows $A \subset B$ and $x \notin B$, but $x \notin A$.

Transitive Reasoning

THE UNDERLYING IDEA

If $A \subset B$ and $B \subset C$, then we can conclude that $A \subset C$.



Premise:	All elements in *A* are also in *B*.	$p \rightarrow q$:	If an element is not in *B*, then it is not in *A*.
Premise:	*x* is not an element in *B*.	p:	*x* is not an element in *B*.
Conclusion:	*x* is not an element in *A*.	$\therefore q$:	*x* is not in *A*.

The argument is VALID because if *A* is in *B* and *B* is in *C*, then we can be 100 percent certain that *A* is in *C*.

But watch out! The diagram to the right shows that if $A \cap B$ and $B \cap C$, it is not necessarily true that $A \cap C$.