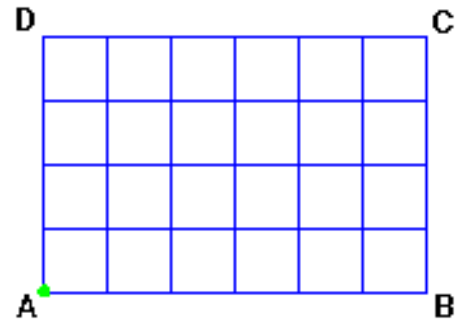


## Paper Pool Game

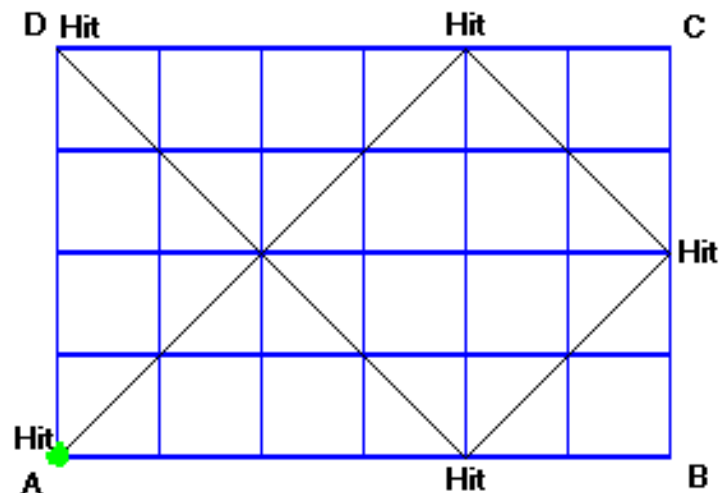
Marisa made up a game called Paper Pool. Her "pool tables" were rectangles drawn on grid paper. The "pockets" at each corner were labeled A (lower left), B (lower right), C (upper right), and D (upper left). Marisa always labeled the corners in this order. To the right is one of her Paper Pool tables. Marisa described the size of a table by giving the length of a horizontal side first and the length of a vertical side second. This Paper Pool table is 6 by 4.



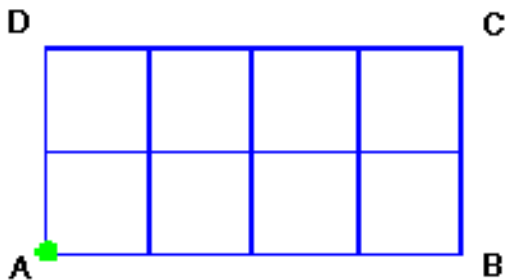
### How to Play Paper Pool

- The ball always starts in corner A.
- To start the ball traveling, it is hit with an imaginary cue (a stick for hitting a pool ball).
- The ball always travels at a  $45^\circ$  diagonal across the grid.
- If the ball hits a side of the table, it bounces off at a  $45^\circ$  angle and continues its travel.
- If the ball hits a pocket, it stops.

Marisa played Paper Pool on the table at the right. The diagonal lines show the ball's path. On this table, the ball stops at corner D and has a total of 5 hits - one as it was struck by the imaginary cue, three as it bounced off the sides, and one when it stopped at pocket D.



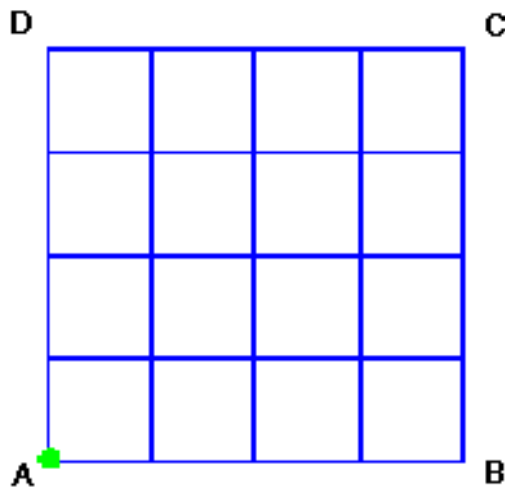
Draw the path the ball would take on the two Paper Pool tables below. For each table, record at what pocket the ball stops, how many hits occurred on its journey, and the dimensions of the table (give the horizontal length first and vertical length second).



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

Dimensions: \_\_\_\_\_

After playing Paper Pool on tables of several sizes, Marisa wondered whether there is a way to predict the pocket at which the ball will stop and how many hits will have occurred. You will try to answer her questions.

## Paper Pool Game (continued)

### Part 1: Investigate Two Questions

Investigate these two questions. Each question is asking you to find out what happens to a ball as it travels on the Paper Pool Tables.

1. In what corner will the ball stop?
2. How many hits will have occurred by the time the ball stops?

The Paper Pool tables on the Labsheets will help you start thinking about the questions. You may need to draw more tables on grid paper to test your ideas about where the ball will stop and how many hits will have occurred. Review "How to Play Paper Pool" before you start.

When you think you can predict the outcomes, write rules that you could use to determine what will happen to the ball as it travels on a table of any size. Your rules should tell you, without drawing the path, the number of hits and the ending corner for the ball.

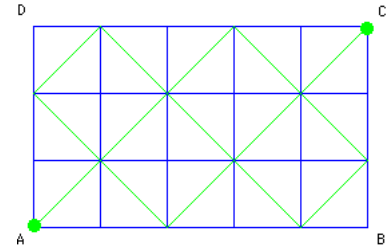
### Part 2: Write a Report

When you have explored several sizes of Paper Pool Tables and have reached some conclusions, write a report on your work. Include the following in your report:

1. A summary of the rules you found, why you think your rules are correct, and anything else you discovered. You might discuss what you noticed as you examined the paths for the different tables and what helped you to arrive at your rules.
2. A drawing of one new Paper Pool table (not from the labsheets) for each rule that demonstrates that the rule accurately predicts what will happen.
3. Your drawings of the ball's path on each Paper Pool Table on the labsheets as well as any other Paper Pool tables you constructed to help you derive your rules.
4. Any tables, charts, or other schemes or tools you used to organize your information and look for patterns.
5. An explanation of any other patterns you found or ideas you have about Paper Pool tables and the path of the ball. (For example, on which tables does the ball's path follow the same basic course? On which tables does the design created by the ball's path look the same?)

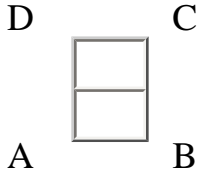
## Extension Question

Can you predict the length of the path the ball will travel on any size Paper Pool table? Each time the ball crosses a square, the distance it travels is 1 diagonal unit. How many diagonal units will the ball's path cover? For example, in the 5 x 3 table to the right, the path length is 15 diagonal units.



Name: \_\_\_\_\_

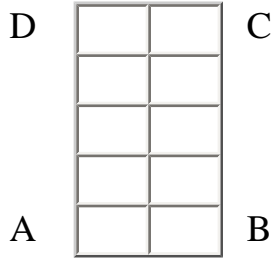
# Paper Pool Tables 1



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

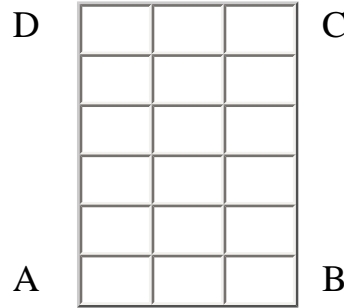
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

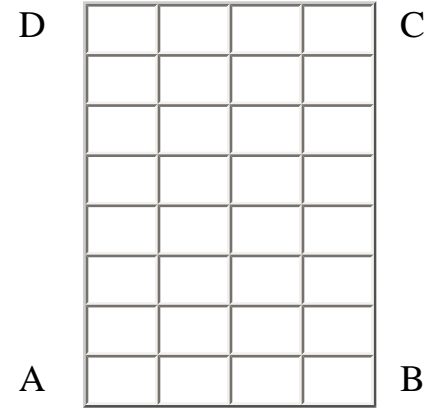
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Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

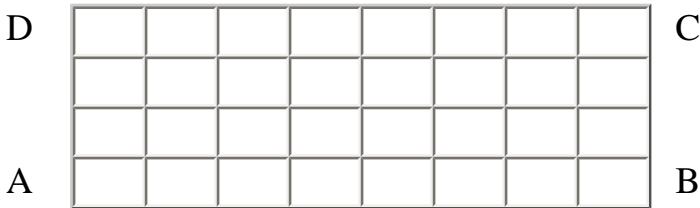
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

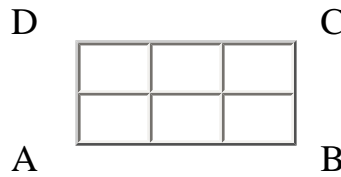
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

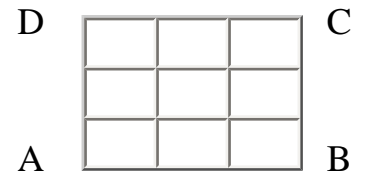
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

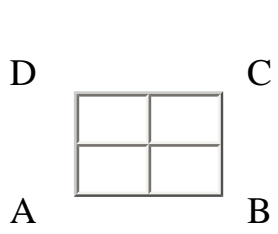
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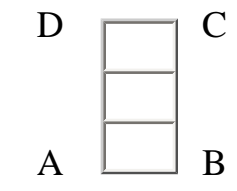
Corner: \_\_\_\_\_

Hits: \_\_\_\_\_

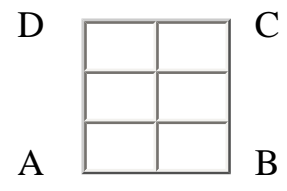
Dimensions: \_\_\_\_\_



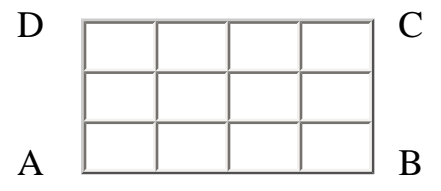
Corner: \_\_\_\_\_  
Hits: \_\_\_\_\_  
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_  
Hits: \_\_\_\_\_  
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_  
Hits: \_\_\_\_\_  
Dimensions: \_\_\_\_\_



Corner: \_\_\_\_\_  
Hits: \_\_\_\_\_  
Dimensions: \_\_\_\_\_

:

Name:

## Record Sheet

Corner where ball stops.	Total Number of Hits Including Start and Finish									
	2	3	4	5	6	7	8	9	10	11
<b>B</b>		4 x 2								
<b>C</b>	4 x 4									
<b>D</b>				6 x 4						



Mark the basic tables with a \*.

What is the rule for the basic tables?

What is the rule for the other tables?

Does the rule for the other tables also work for basic tables?

How many squares would be crossed in a 24 x 36 table?

**Name:**

## Record Sheet

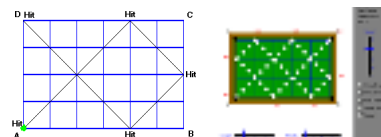
Corner where ball stops.	Total Number of Hits Including Start and Finish									
	2	3	4	5	6	7	8	9	10	11
<b>B</b>		4 x 2								
<b>C</b>	4 x 4									
<b>D</b>				6 x 4						

Name:

## How Many Squares Are Crossed?

There is also a rule to predict how many squares the path crosses on a paper pool table.

For example: This path crosses 12 squares. Use your Paper Pool worksheets or the Paper Pool Software and record the data from each table.



Size of Table	Number of Squares Crossed

Size of Table	Number of Squares Crossed

Mark the basic tables with a \*.

What is the rule for the basic tables?

What is the rule for the other tables?

Does the rule for the other tables also work for basic tables?

How many squares would be crossed in a 24 x 36 table?

# Sample Record Sheet

Corner where ball stops.	Total Number of Hits Including Start and Finish									
	2	3	4	5	6	7	8	9	10	11
<b>B</b>		2 x 1 4 x 2		2 x 3 4 x 1 4 x 6		2 x 5 4 x 3 6 x 1		2 x 7 4 x 5 8 x 10		2 x 9 4 x 7 6 x 5 8 x 3 10 x 1
<b>C</b>	1 x 1 2 x 2 3 x 3 4 x 4		1 x 3 3 x 1 2 x 6		1 x 5 5 x 1 10 x 2		1 x 7 5 x 3 3 x 5 7 x 1 6 x 10		1 x 9 3 x 7 7 x 3 9 x 1	
<b>D</b>		1 x 2 2 x 4 3 x 6 4 x 8		1 x 4 3 x 2 6 x 4		1 x 6 3 x 4 5 x 2		5 x 4 7 x 2		1 x 10 3 x 8 5 x 6 7 x 4 9 x 2

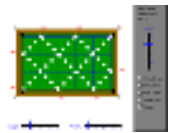
See the page of [sample rules](#) for the generalizations that can be made from this table.

## i-Math Investigation / 6 - 8

### Analyzing Numeric and Geometric Patterns of Paper Pool Part Four - Go the Distance

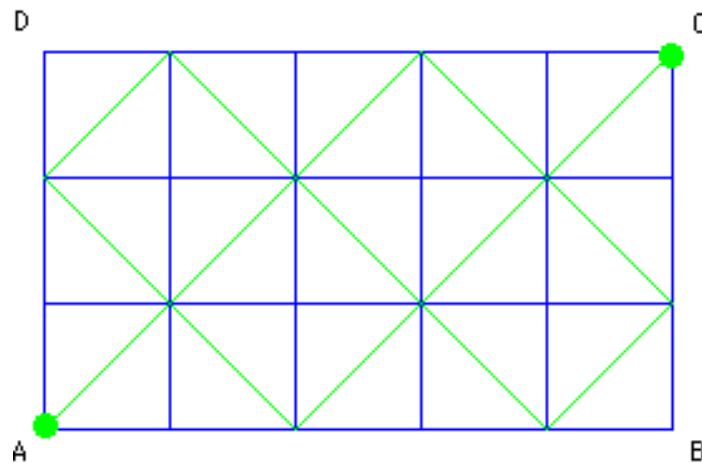
There are other questions that you can explore for the Paper Pool game. Here are two more questions for possible investigation.

Examine the path of the ball for several pool tables using the Paper Pool table software or by hand using grids of your own.



1. Can you predict the length of the path the ball will travel on any size Paper Pool Table? Each time the ball crosses a square, the distance it travels is 1 diagonal unit. How many diagonal units will the ball's path cover?

#### 5 x 3 Paper Pool Table



**Path Length = 15 diagonal units**


2. What kind of symmetry does the path of the ball have for different pool tables?

For what dimensions of pool tables, does the path have a vertical line of reflection?

For what dimensions of pool tables, does the path have a horizontal line of reflection?

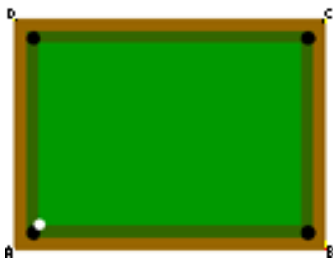
For what dimensions of pool tables, does the path have a rotational symmetry?

## Write a Report

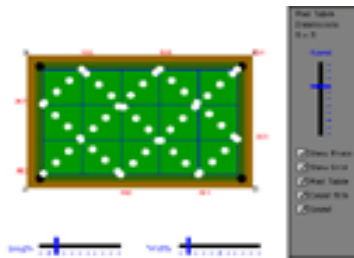
Thoughts  
for  
Teachers 

When you have explored several sizes of Paper Pool tables and have reached some conclusions, write a report on your work. Include the following in your report:

1. A summary of the rules you found, why you think your rules are correct, and anything else you discovered. You might discuss what you noticed as you examined the paths for different tables and what helped you to arrive at your rules.
2. A drawing of one Paper Pool table for each rule that demonstrates that the rule accurately predicts what will happen.
3. Any tables, charts, drawings or other tools you used to organize your information and look for patterns.
4. An explanation of any other patterns you found or ideas you have about Paper Pool tables and the path of the ball.



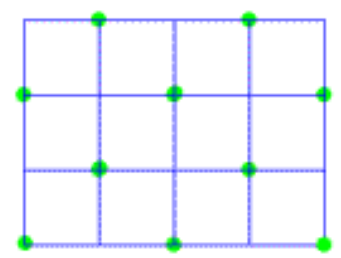
[Paper Pool Game](#)



[Explore More Tables](#)

Data			
	A	B	C
1	Dimension	# of Hits	Corner
2	6 x 4	5	D
3	4 x 4	2	C
4	4 x 2	3	B
5			

[Look for Patterns](#)



[Go the Distance](#)

[Back to the beginning of this i-Math Investigation](#)

[References](#)



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