Selected Answers - Work to a Conclusion

Activity 1

1. Responses will vary depending upon the data set and corresponding function used.
   - For the Large Whelk Data: N = 1 + 20.4/(H - 0.84), the minimum is near H = 4.98 and W = 29.5.
   - For the Sample Peanut Data with N = 1 + 59.5/(H - 13.8), the minimum is near H = 42.5 and W = 131.

   For nearby heights, the amount of work is nearly constant. Thus, in this range of heights other factors may contribute to the ìoptim alî height.

2. These values are within the range obtained from examining the numerical values of work produced from the data. Since the data has errors in H and W, the symbolic result would be less impacted by slight errors. However, either one could be used.

4. Students may not know about oblique asymptotes. The work function provides an excellent opportunity to discuss oblique asymptotes. See the Reflection Activity.
   - For the Large Whelk Data: N = 1 + 20.4/(H - 0.84) and W = H ◊ N, there is a vertical asymptote at 0.84. N has a horizontal asymptote at 1, and W has an oblique asymptote of W = H + 20.4.
   - For the Sample Peanut Data with N = 1 + 59.5/(H - 13.8) and W = H ◊ N, there is a vertical asymptote for both at 13.8, N has a horizontal asymptote at 1, and W has an oblique asymptote of W = H + 59.5.

Activity 2

1. The standard form provides information about the degrees of the polynomials involved. Factored form is the best for finding zeros and vertical asymptotes. Proper fraction form is useful for determining oblique and horizontal asymptotes as well as the long term behavior of the function.

2. In each case the work done for very large heights is essentially the height plus some constant since the fractional term will be near 0. This says that the work done in dropping an object will be nearly the work done in lifting the object to the height. This makes sense since the number of drops will be near 1.

Reflection Questions

The following are quick responses. Students could be asked to summarize the entire investigation in a report. Specifically, students could be asked to focus the report on the first question. Are the crows dropping the whelk from an optimal height? Students should use the Large Whelk Data provided.

Answers to the first two reflection questions are provided below.

1. Using the large whelk data, the optimal height is around 5 meters. Reto Zach used this fact to argue that the crows are dropping the whelk at the optimal height and serves as an example of optimal foraging. Given the large range of heights for which the work is nearly equal, other influences may be at work. For example, the crows may use a lower height so that there is less chance of losing their work to another bird or to be more accurate in where the whelk is dropped.

2. Lifting half an object twice would require the same amount of work as lifting the whole object. Using similar reasoning, the work in lifting an object a number of times to the same height would be: Work = weight ◊ height ◊ number of drops. Since the weight enters only as a scalar constant, the location height of the minimum work would remain the same.

Knowledge of rational functions helped to make sense of the potential asymptotes involved. To find a model using symbolic methods, some assumption of the horizontal asymptote was needed. Knowledge of rational functions helps to make sense of the work equation as well. The oblique asymptote of the work function reflects our understanding that the work in lifting an object once varies directly with the height to which it is lifted.