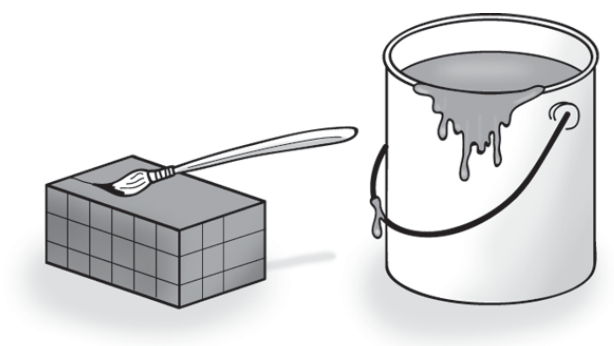
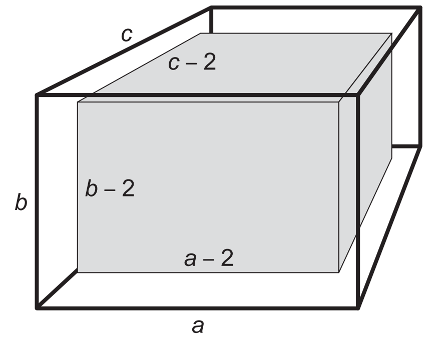
This brainteaser was written by Derrick Niederman.

A rectangular wooden block (not necessarily a cube) is painted on the outside and then divided into one-unit cubes. It turns out that exactly half of the cubes have paint on them. What were the dimensions of the block before it was painted?



Solution: Many different blocks match this criteria, including **5 × 13 × 132**, **5 × 14 × 72**, **5 × 15 × 52**, **5 × 16 × 42**, **5 × 17 × 36**, **5 × 18 × 32**, **5 × 20 × 27**, **5 × 22 × 24**, **6 × 9 × 56**, **6 × 10 × 32**, **6 × 11 × 24**, **6 × 12 × 20**, **6 × 14 × 16**, **7 × 7 × 100**, **7 × 8 × 30**, **7 × 9 × 20**, **7 × 10 × 16**, **8 × 8 × 18**, **8 × 9 × 14**, and **8 × 10 × 12**.



When the block is divided, only those cubes with a face on the surface of the block will be painted. All interior cubes will be unpainted. If the edges of the cube are *a*, *b*, and *c*, then *abc* cubes will be painted and (a – 2)(b – 2)(c – 2) cubes will be unpainted.

Since half of the cubes are painted, this leads to the following equation:

$$\tfrac{1}{2}abc = (a-2)(b-2)(c-2)$$

You can now implement a guess-and-check strategy. Substitute values for *a* and *b* into the equation, and solve for *c*. If you find a positive integer value for *c*, you have a solution to the puzzle. If not, try again. For instance, the two examples below use $a = 4$ and $b = 5$ on the left and $a = 6$ and $b = 9$ on the right.

$$\begin{aligned} \tfrac{1}{2}(4)(5)c &= (4-2)(5-2)(c-2) \\ 10c &= 6c - 12 \\ 4c &= -12 \\ c &= -3 \end{aligned}$$

c is negative
not a solution

$$\begin{aligned} \tfrac{1}{2}(6)(9)c &= (6-2)(9-2)(c-2) \\ 27c &= 28c - 56 \\ -c &= -56 \\ c &= 56 \end{aligned}$$

c is a positive integer
dimensions are 6 × 9 × 56

The method above gives one possible solution. Finding **all** possible solutions requires a bit more work. Solving the original equation above for *a* yields:

$$\begin{aligned} \tfrac{1}{2}abc &= (a-2)(b-2)(c-2) \\ abc &= 2(2abc - 2ab - 2ac - 2bc + 4a + 4b + 4c - 8) \\ abc - 4ac - 4ab + 8a &= 4bc - 8b - 8c + 16 \\ a &= \frac{4bc - 8b - 8c + 16}{bc - 4b - 4c + 8} \end{aligned}$$

Having chosen values for *b* and *c*, you can determine the corresponding value of *a*. This can be messy, so a spreadsheet program can be helpful with the calculations. It is also helpful to observe that at least one of the dimensions must be less than 10 units. If all the dimensions are greater than 10 units, the number of interior cubes will always be greater than half the total number. Knowing this means that you do not need to consider blocks whose dimensions are all greater than 10.