



*This brainteaser was written by Julia Zurkovsky.*

How do I love thee? Let me graph the ways!

Can you come up with one or more equations to graph a heart on the coordinate plane? The equations can be rectangular, polar, or parametric.

**Bonus:** Can you shift your heart so the graph or its interior includes the point  $(2, 14)$ ?



**Solution: Answers will vary.**

One way to create a heart is to use the graphs of two ellipses and restrict the domain.

$$2x^2 - 2xy + y^2 - 1 = 0 \quad \{x | x \geq 0\}$$

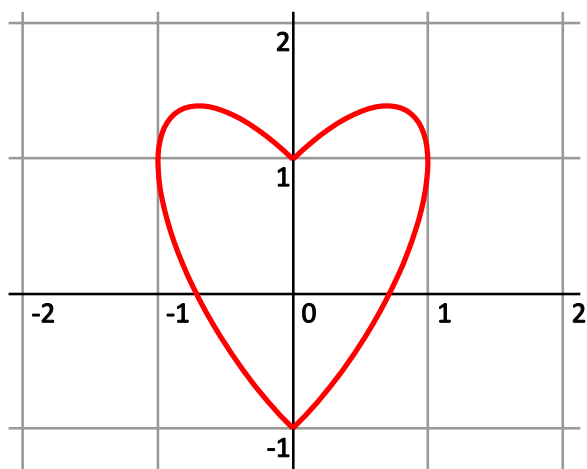
$$2x^2 + 2xy + y^2 - 1 = 0 \quad \{x | x \leq 0\}$$

Alternatively, you can solve for  $y$  and use the absolute value function, and then restricting the domain is unnecessary.

$$y = |x| + \sqrt{1 - x^2}$$

$$y = |x| - \sqrt{1 - x^2}$$

In either case, the result is a heart curve that looks like this:



To move heart so that it covers (2,14), adjust the absolute value equations as follows:

$$y = |x - 2| + \sqrt{1 - (x - 2)^2} + 14$$

$$y = |x - 2| - \sqrt{1 - (x - 2)^2} + 14$$

The following polar equation will yield a cardioid:

$$r = 1 - \sin \theta$$

Other heart-shaped graphs can be created using polar, parametric or rectangular equations. A number of examples can be found at <http://mathworld.wolfram.com/HeartCurve.html>.