

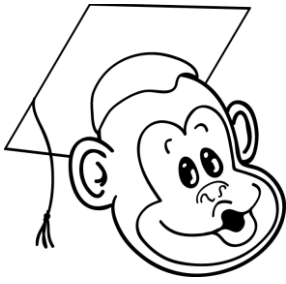


This brainteaser was written by Patrick Vennebush.

A magic rectangle is an $m \times n$ array of the positive integers from 1 to $m \times n$ such that the numbers in each row have a constant sum and the numbers in each column have a constant sum (although the row sum need not equal the column sum). Shown below is a 3×5 magic rectangle with the integers 1-15.

6	7	8	9	10
13	3	1	11	12
5	14	15	4	2

Two of three arrays below can be filled with the integers 1-24 to form a magic rectangle. Which one can't, and why not?



Solution: The first array, which has 3 rows and 8 columns, cannot be filled to create a magic rectangle.

The integers 1-24 have a sum of 300. This sum can be determined by noticing that the numbers can be regrouped as 12 pairs of numbers with a sum of 25 each:

$$(1 + 24) + (2 + 23) + (3 + 22) + \dots + (12 + 13) = 300$$

In general, the sum of the first n positive integers is given by the following formula:

$$s(n) = \frac{(n)(n+1)}{2} = \frac{n^2 + n}{2}$$

Because the sum of the integers 1-24 is 300, to be placed in the 3×8 array to form a magic rectangle, the sum of each row would have to be $300 \div 3 = 100$. This can be done in a number of ways. However, the sum of each column would have to be $300 \div 8 = 37.5$, which is impossible.

In general, a magic rectangle can be created only if the number of rows and number of columns are both even or both odd; that is, a magic rectangle cannot be created if one is even and the other is odd.

The other arrays can be filled in many different ways. One example of each is shown below.

1	2	3	22	23	24
19	20	21	4	5	6
18	17	16	9	8	7
12	11	10	15	14	13

1	23	3	21	5	19	7	17	16	15	11	12
24	2	22	4	20	6	18	8	9	10	14	13