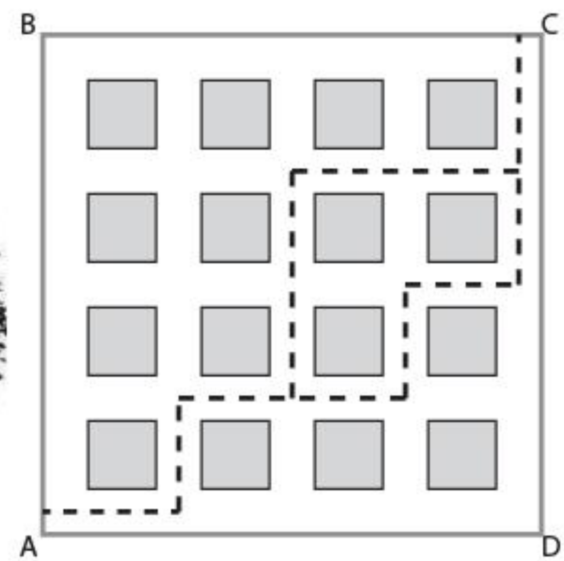
Puzzle provided by Kordemsky: The Moscow Puzzles (Dover)

“In our Mathematics Circle we diagramed 16 blocks of our city. How many different routes can we draw from *A* to *C* moving only upward and to the right?”

Different routes may, of course, have portions that coincide (as in the diagram).

“This problem is not easy. Have we solved it by counting 70 different routes?”

What answer should we give these students?



Solution:

Yes, you will get mixed up if you try to draw each route from *A* to *C* – it is too complicated. It is simper to solve the problem for points near *A* and progress point by point to *C*. The diagram labels all the points from 1*a* (which is *A*) to 5*e* (which is *C*).

It is evident that there is only 1 route from *A* to the nearest points on *AB* and *AD* (2*a* and 1*b*). You can get to 2*b* through either of these points (2 routes). Now the crossing 2*c* can be reached from 2*b* (2 routes) or from 1*c* (1 more route, making 3 in all). Analogously, there are 3 routes to 3*b*.

Now it becomes clear that the number of routes shown in each crossing is the sum of the number of routes shown immediately to the left and immediately below—which is logical, because all moves are up or to the right.

We keep adding, working from point to point, until we reach *C* with its 70 different routes from *A*.