

# Seeing Music, Hearing Waves

NAME \_\_\_\_\_

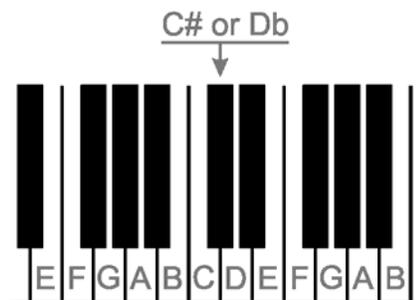
In this activity, you will calculate the frequencies of two octaves of a chromatic musical scale in standard pitch. Then, you will experiment with different combinations of notes and related sine waves to observe why some combinations of musical notes sound harmonious and others have a dissonance. When you check out note combinations, you will listen to those combinations played on a keyboard to associate the sounds with sine waves.

## The Chromatic Scale — A Geometric Series

Musical pitches (notes) are determined by their frequency, which is measured in vibrations per second, or Hertz (Hz). The notes on a piano keyboard form a *chromatic scale*.

A chromatic scale divides the octave into its semitones. There are twelve semitones, or half steps, to an octave in the chromatic scale.

The white keys on a keyboard are A, B, C, D, E, F, and G. The black keys are named relative to their adjacent white keys. For example, the black key between the C and D keys is known as either *C sharp (C#)* or *D flat (Db)*.



The frequencies of the notes that make up a chromatic scale form a geometric sequence where each term has the form  $ar^n$ . The ratio that generates the chromatic scale is  $r = 2^{1/12}$  or  $r = \sqrt[12]{2}$ .

1. The A note below middle C on a keyboard has a frequency of 220 Hz. Using this value, calculate the frequencies of the terms that generate a two-octave chromatic scale.

Calculate each value using the original value of  $a = 220$  and the formula  $ar^n$ , where  $r = 2^{1/2}$ . Using a scientific calculator, this can be accomplished by the following steps:

- Enter  $220$ , and hit  $\text{ENTER}$ .
- Enter  $\times 2 \wedge (1 \div 12)$ . Hit  $\text{ENTER}$ .
- Repeatedly hit  $\text{ENTER}$ . Each time  $\text{ENTER}$  is pressed, the value for the next row will be calculated.

Round each frequency to the nearest whole number.

LOWER OCTAVE	FREQUENCY (Hz)
A	220 Hz
A# or Bb	$220(2^{1/2})^1 \approx 233$
B	$220(2^{1/2})^2 \approx$
C	$220(2^{1/2})^3 \approx$
C# or Db	
D	
D# or Eb	
E	
F	
F# or Gb	
G	
G# or Ab	

HIGHER OCTAVE	FREQUENCY (Hz)
A	
A# or Bb	
B	
C	
C# or Db	
D	
D# or Eb	
E	
F	
F# or Gb	
G	
G# or Ab	

\* This note is known as "Middle C," because it appears near the middle of a piano keyboard.

2. In the table above, compare the frequencies of notes that are one octave apart. For instance, compare A in the lower octave (left column) with A in the higher octave (right column), compare A# in the lower octave with A# in the higher octave, and so forth. How do frequencies an octave apart appear to be related?

The sine wave related to a musical pitch has the following form, where  $A$  is the *amplitude* of the sound (or the volume, measured in decibels) and  $B$  is the frequency of the note (measured in Hz):

$$f(x) = A \sin(Bx)$$

3. Based on the frequencies in the above table, write the sine functions to represent both the low and high octaves for the C notes. (The value of  $A$  represents the volume of the note, so any value can be used. For the remainder of this activity sheet, let  $A = 2$ .)

Then, graph the sine function for each note on your graphing calculator, and change the viewing window to show two cycles of the curve. (To do this, set  $X_{\min} = 0$ , and set  $X_{\max}$  to twice the value of the period; the period is equal to  $2\pi$  divided by the frequency.) Graph the sine waves for notes in both octaves in the same viewing window. Draw the graph, and record the scale, frequency and period below.

**C Notes**

Xmin: \_\_\_\_\_

Xmax: \_\_\_\_\_

Ymin: \_\_\_\_\_

Ymax: \_\_\_\_\_

*Middle C*

Frequency: \_\_\_\_\_ Period: \_\_\_\_\_

*Lower C*

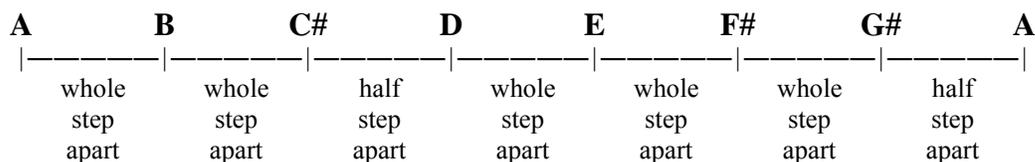
Frequency: \_\_\_\_\_ Period: \_\_\_\_\_



## For Group 1

Adjacent notes in a chromatic scale are half-steps apart. A **diatonic** musical scale (also called a **major** scale) is an eight-note sequence of notes that are both half- and whole steps apart, as shown below.

The eight notes of the A-major scale are:



6. A major chord (or **triad**) of any scale consists of the *first*, *third*, and *fifth* notes of the scale. Based on the A major scale identified above, identify the notes of the A major chord, the frequencies of those notes, the associated sine function with  $A = 2$ , and the period of the sine wave.

NOTE	NAME	FREQUENCY ( $B$ )	SINE FUNCTION $f(x) = 2 \sin(Bx)$	PERIOD OF SINE WAVE
First note of the A major scale	A	220 Hz		
Third note of the A major scale				
Fifth note of the A major scale				

7. When all three of the above sine waves are graphed, they intersect at the point  $(0, 0)$ .
- a. What are the coordinates of the second point where all three sine waves intersect? (The first point of intersection in the window should be the origin.)

( \_\_\_\_\_ , \_\_\_\_\_ )

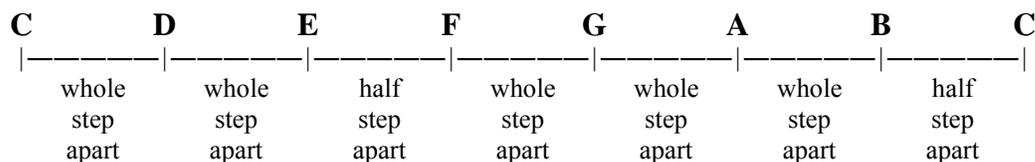
- b. From the origin to the next point of intersection, record the number of cycles for each of the sine waves.

NOTE OF THE CHORD	NUMBER OF CYCLES
First note of the scale	
Third note of the scale	
Fifth note of the scale	

## For Group 2

Adjacent notes in a chromatic scale are half-steps apart. A **diatonic** musical scale (also called a **major** scale) is an eight-note sequence of notes that are both half- and whole steps apart, as shown below.

The eight notes of the C-major scale are:



8. A major chord (or **triad**) of any scale consists of the *first*, *third*, and *fifth* notes of the scale. Based on the C major scale identified above, identify the notes of the C major chord, the frequencies of those notes, the associated sine function with  $A = 2$ , and the period of the sine wave.

NOTE	NAME	FREQUENCY ( $B$ )	SINE FUNCTION $f(x) = 2 \sin(Bx)$	PERIOD OF SINE WAVE
First note of the A major scale	C	262 Hz		
Third note of the A major scale				
Fifth note of the A major scale				

9. When all three of the above sine waves are graphed, they intersect at the point  $(0, 0)$ .
- a. What are the coordinates of the second point where all three sine waves intersect? (The first point of intersection in the window should be the origin.)

( \_\_\_\_\_ , \_\_\_\_\_ )

- b. From the origin to the next point of intersection, record the number of cycles for each of the sine waves.

NOTE OF THE CHORD	NUMBER OF CYCLES
First note of the scale	
Third note of the scale	
Fifth note of the scale	

## For Group 1 and Group 2

**10.** Compare your findings regarding the notes making up the A major scale and the notes making up the C major scale. Then, experiment with other major scales that start with other notes. Do the sine waves of the triad (the first, third, and fifth notes) compare in the same way for those scales? Summarize your findings below.

**11.** Experiment with other note combinations. Graph the sine wave of two notes that are half-steps apart, such as A and A#. How many cycles do you have to graph before the two curves intersect? Then, have a musician play A and A# together to hear how they sound. Would you describe the sound as harmonious or dissonant?

Similarly, compare sine waves for three notes that are half-steps apart, such as B, C, and C#, and listen to those notes played together.

In the space below, describe what dissonance looks like and what harmony looks like in terms of sine waves.