Modeling Orbital Debris Problems   NAME ________________________

Space Debris: Is It Really That Bad?

One problem with which NASA and space scientists from other countries must deal is the accumulation of space debris in orbit around Earth. Such debris includes payloads that are no longer operating; spent stages of rockets, assorted parts and lost tools; debris from the breakup of larger objects or from collisions between objects; and countless small pieces, such as flakes of paint and even smaller objects. Because bodies in Earth orbit travel at approximately 17500 miles per hour, a collision with even a tiny object can have catastrophic effects. In 1990, scientists estimated that a total of 4 million pounds of debris was in Earth orbit. They also estimated that at that time, we were adding 1.8 million pounds per year to the already serious problem, which in a few years would result in 9.5 million pounds of orbital debris. The 1990 prediction also stated that the amount of debris being added was anticipated to increase to a rate of 2.7 million pounds per year by the year 2000.

1. **a)** How much is 4 million pounds of anything? Or 9.5 million?

b) Give at least three concrete examples that would help another person get a sense of how much millions of pounds of debris is. For example, finish the following sentence.

c) A total of 9.5 million pounds of pennies would fill __________________.

Modeling the Problem: Linear Growth

The problem of determining the amount of debris in space and the anticipated rate of increase of such matter is not one that can be solved directly. We cannot locate, count, and weigh all objects in orbit. Nor can we predict with assurance when two of them will collide. Instead, we must rely on mathematical models to help us represent the problems and identify trends and expected outcomes. In these activities, you will create and compare various mathematical models to help you investigate some of the questions raised by the proliferation of orbital debris. These models are greatly simplified in their assumptions so that you can investigate them with calculators, spreadsheets, and graphing utilities, but they provide insight into the process of mathematical modeling and its importance.

1. When creating models, mathematicians favor the simplest model that will account for the phenomena in question. Generally, a linear model gives the simplest case. So, using the reported 1990 rate of increase of 1.8 million pounds per year and assuming 4 million pounds of existing debris at the beginning of 1990, write a linear model to predict the number of pounds of orbital debris at the end of any given year, t. Assume that $t = 1$ represents 1990.

2. Write a second linear model using the predicted 2.7 million pounds per year rate of increase and the initial 4 million pounds for 1909.

3. Evaluate each model for several years to determine the year in which the predicted 9.5 million pounds of accumulated debris would occur

a) With the first model:

b) With the second model:

4. When a vehicle travels at a constant rate, r, for a length of time, t, the distance traveled by the vehicle is modeled by a linear function. Compare the familiar linear model for distance-rate-time with your linear models for accumulating space debris. Why can we refer to your linear models as "constant-velocity models for amassing space debris"?

5. Do you think that either of your linear models accurately represents that situation of escalating amounts of space debris as described in the original paragraph? Why or why not?

Refining the Model: Quadratic Growth

Does either rate, 1.8 million pounds per year or 2.7 million pounds per year, tell us how much debris is building up between 1990 and 2000? Which rate of increase should we use? Obviously the amount being added each year is changing during this period, but by how much each year? The problem is one of acceleration, no constant velocity, so we need to adjust our model.

Again, let's make the simplest assumption: the rate at which we are adding debris increases at a constant rate from 1.8 million pounds per year in 1990 to 2.7 million pounds per year in 2000. This change means that over the ten-year period from the end of 1990 through 2000, the rate (velocity) of littering will increase by 0.9 million pounds per year (2.7 - 1.8 = 0.9), and we are making the assumption that this increase is achieved in equal annual increments of 0.09 million pounds per year in each year of the decade.

1. Complete the following table to show the amount of debris added each year and the total amount in orbit at the end of the year.

YEAR	AMOUNT ADDED IN YEAR (MILLIONS OF POUNDS)	TOTAL IN ORBIT AT END OF YEAR (MILLIONS OF POUNDS)
1990	1.8	5.8
1991		
1992		
1993		
1994		
1995		
1996		
1997		
1998		
1999		
2000	2.7	

2. Since we assumed that the increase in the velocity of littering was achieved in equal annual increments, you can write a linear equation that describes the increase in the amount of debris being added each year (i.e, the increase in the annual velocity of littering as a function of the number of years since 1990. In this case, we let a = 0 in 1990 because we are assuming that the 1990 rate of 1.8 million pounds per year is our baseline rate. Then d = f(a) represents the rate of littering a years after 1990.

3. The situation described in your equation, where the rate of increase of litter is itself increasing at a constant rate, is analogous to a vehicle that accelerates at a constant rate from an initial velocity, v0, to a final velocity, vf. Use the data generated in the foregoing table to create a scatterplot of the total number of pounds of orbital debris that have accumulated relative to the year. You graph should cover the period from 1990 through 2000.

4. Fit a line to you data and decide whether the accumulation of debris appears to be linear. Write your conclusion and describe the evidence on which you based your decision.

5. Using a graphing calculator or computer graphing program, calculate the linear-regression equation for these data.

a) Do your calculations support a linear relationship? Explain.

b) How does this line compare with the line that you fitted manually?

6. Next generate a quadratic-regression equation for the same data.

a) Write the quadratic-regression model here:

b) How well does this equation fit the data compared with the linear approximation?

7. Compare your quadratic-regression equation with the two linear-regression equations that you developed earlier.

8. In each case, use your models to predict the accumulation of debris after twenty years, thirty years, and fifty years. Describe the behavior of the linear model versus the quadratic model over time.

9. For the period from 1990 to 2000, the graph of the quadratic model lies between the graphs of the two linear models.

a) Explain why this result is reasonable.

b) Will the quadratic graph always lie between the two linear graphs? Explain.

10. Explain why the quadratic model for the debris problem can be described as a "uniform acceleration" model.

One More Perspective: Exponential Growth

So far, you have looked at two models, a "constant velocity" linear model and a "uniformly accelerated" quadratic model. Let's look at one more model.

1. Suppose that the amount of litter added each year grew not by a fixed number of pounds but by a fixed percent of the amount already in space - a situation analogous to an investment of money with interest compounded annually. For example, what would happen to the original 4 million pounds if the litter added each year was 20 percent of the amount already in orbit? Complete the following table to determine the amount of debris that would accumulate over the period from 1990 to 2000. A spreadsheet is recommended for this activity.

YEAR	AMOUNT ADDED EACH YEAR EQUALING 20% OF PREVIOUS AMOUNT (MILLIONS OF POUNDS)	TOTAL IN ORBIT AT END OF YEAR (MILLIONS OF POUNDS)
(1989)	(n/a)	4
1990	0.8	4.8
1991	0.96	
1992		
1993		
1994		
1995		
1996		
1997		
1998		
1999		
2000		

2. Write an exponential model to describe the growth of the original 4 million pounds of debris over the years:

3. Use your model to predict the amount of debris that would accumulate in twenty years, thirty years, and fifty years.

4. Economists use what is referred to as the "rules of 72" to predict how long it will take an amount of money to double if it is invested at a rate of R percent compounded annually. According to the rules of 72, the doubling time, *D*, is given by the equation $D = 72/R$.

a) Use the rule of 72 to predict how long it will take for the amount of debris to double if littering compounds at the rate of 20 percent per year. How long will it take for the original 4 million pounds to increase to 32 million pounds?

b) Do the data in your table agree with those calculations?

5. If the growth of space debris was following an exponential model and concern arose that it would take only twelve years to double the amount of space debris, what must the annual percent increase in debris have been to result in this doubling time of twelve years?

What Goes Up Might Come Down: Extending the Models

All the models you have developed thus far assume that the additional every year some of the debris slows down enough to re-enter the atmosphere where it burns up or, on rare occasions, returns to Earth. Assume for the moment that 10 percent of the debris in orbit at the beginning of any year will be destroyed during that year.

1. Modify you linear, quadratic, and exponential models to account for the situation in which additional debris is being added each year while 10 percent of what was already in orbit is being destroyed.

2. In which case – linear, quadratic, or exponential – does the assumption of a 10 percent re-entry rate have the greatest effect?

3. Assume the same rates of adding debris as you did you when you generated the models, but try different rates of annual destruction of orbital debris. In each situation, does a destruction rate exist that will result in a net decrease in orbital debris despite the fact that additional debris is being added?

4. What might be some advantages of knowing if such a rate is possible?

Putting Your Models to Work

The power of mathematical models comes from their ability to enable us to ask "What if?" questions. An earlier question is an example: What if we could increase the rate at which orbital debris is destroyed? Other questions might include these: What if we decrease the rate at which we are adding debris and find a way to increase the rate at which existing debris is destroyed? Such questions lead to open-ended investigations using mathematical models.

1. Working with a partner or a small group, generate two or three specific questions that you would like to investigate.

  a) What if…

  b) What if…

  c) What if…

2. Describe a plan for investigating your questions using spreadsheets, graphing utilities, or other appropriate technology.