Seeing Music, Hearing Waves

SELECTED ANSWERS AND SOLUTIONS

1. The table should be completed as follows.

LOWER OCTAVE	FREQUENCY (HZ)
A	220 Hz
A# or Bb	$220\left(2^{1/12}\right)^{1} \approx 233$
B	$220\left(2^{1/12}\right)^{2} \approx 247$
C	$220\left(2^{1/12}\right)^{3} \approx 262$
C# or Db	277
D	294
D# or Eb	311
E	330
F	349
F# or Gb	370
G	392
G# or Ab	415

HIGHER OCTAVE	FREQUENCY (HZ)
A	440
A# or Bb	466
B	494
C	523
C# or Db	554
D	587
D# or Eb	622
E	659
F	698
F# or Gb	740
G	784
G# or Ab	831

2. The frequency of the note in the higher octave is double the frequency the note of the lower octave.

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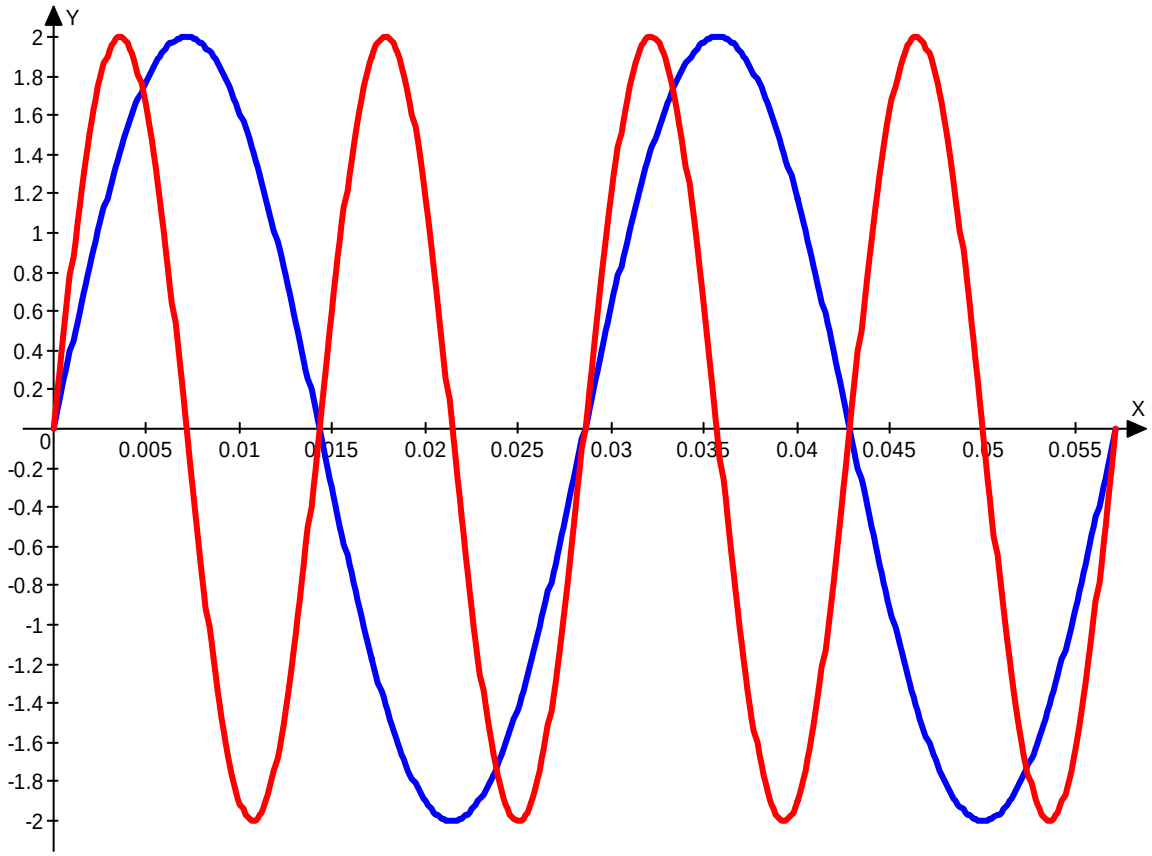
3. A graph similar to the one shown below will appear when $y = 2\sin(220x)$ and $y = 2\sin(440x)$ are graphed in a window with Xmin = 0, Xmax = 0.057, Ymin = -2, and Ymax = 2.

Middle C
Frequency: 523 Period: $2\pi \div 523 \approx 0.012$

Lower C
Frequency: 262 Period: $2\pi \div 262 \approx 0.024$



6. The table should be completed as follows:

NOTE	NAME	FREQUENCY (B)	SINE FUNCTION $f(x) = 2\sin(Bx)$	PERIOD OF SINE WAVE
First note of the A major scale	A	220 Hz	$f(x) = 2\sin(220x)$	0.02856
Third note of the A major scale	C#	277 Hz	$f(x) = 2\sin(277x)$	0.02268
Fifth note of the A major scale	E	330 Hz	$f(x) = 2\sin(330x)$	0.01910

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7. The graphs first intersect at the origin (0, 0).

a. (0.29, 0)

b. The number of cycles for each sine wave is given below.

NOTE OF THE CHORD	NUMBER OF CYCLES
First note of the scale	2
Third note of the scale	2.5
Fifth note of the scale	3

8. The table should be completed as follows:

NOTE	NAME	FREQUENCY (B)	SINE FUNCTION $f(x) = 2\sin(Bx)$	PERIOD OF SINE WAVE
First note of the A major scale	C	262 Hz	$f(x) = 2\sin(262x)$	0.0240
Third note of the A major scale	E	330 Hz	$f(x) = 2\sin(330x)$	0.0190
Fifth note of the A major scale	G	392 Hz	$f(x) = 2\sin(392x)$	0.0160

9. Again, all three graphs first intersect at the origin (0, 0).

a. (0.048, 0)

b. The number of cycles for each sine wave is given below.

NOTE OF THE CHORD	NUMBER OF CYCLES
First note of the scale	2
Third note of the scale	2.5
Fifth note of the scale	3

10. The three curves will intersect at the origin and then again when the first note completes two cycles. This will coincide with the point when the third note completes two-and-a-half cycles and when the fifth note completes three cycles.

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