Voting – Activity Sheet 3

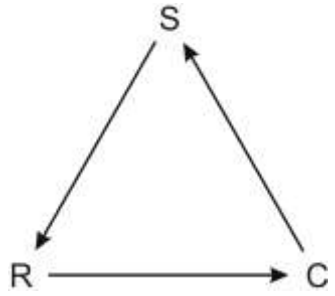
NAME _____

DATE:_____

Tournament Digraphs and Condorcet Winners

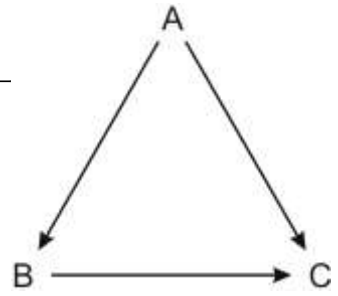
We can use a diagram called a tournament digraph to illustrate the expected results of head-to-head polls among the candidates. We use vertices (dots) to represent each of the candidates and draw an arrow from one candidate to another if the first candidate would beat the second in a head-to-head competition. If no ties occur, exactly one arrow would join every pair of candidates.

Example: The tournament digraph corresponding to the Adventure Club example on the first sheet is sketched to the right.



Note that in the head-to-head matches, skiing beats rafting, rafting beats caving and caving beats skiing.

In the example above, no activity defeated all the other activities in head-to-head voting. Suppose instead that the tournament digraph for a set of preferences looked like this:



1. Who would you expect to win using plurality?___
Hare system? _____
Borda count? _____
Sequential pairwise voting? _____

2. Sketch the tournament digraph for the election on the Strategic Voting page. Does this result agree with your response in question 1 on this page?

3. How many arrows are there in a digraph with five candidates?

With ten candidates?

With n candidates?

4. Sketch the tournament digraph of the following:

	Number of Voters			
Ranking	**38%**	**29%**	**24%**	**10%**
First Choice	W	Z	X	X
Second Choice	X	Y	Z	W
Third Choice	Y	W	Y	Y
Fourth Choice	Z	X	W	Z

5. Determine the winner of the election in problem 4 above, when the Borda count is used.

Is it what you expected?

Note that with four candidates, each first-place vote will be worth 3 points, each second-place votes will be worth 2 points and each third-place vote is one point.

When a candidate beats every other candidate in head-to-head contests, we call the candidate a ***Condorcet winner***.

6. Develop a table of preference lists for which the Condorcet winner would lose the election under the Hare system of voting.