Families ASK

Take-Home Page

Families often ask about basic skills:

Why aren't students learning to add, subtract, multiply, and divide like we did?

Consider the following reply:

In today's middle schools, much more emphasis is placed on the *meaning* of number operations, geometry, statistics, and so forth, than it once was. Students are still being taught to add fractions and to find the percent of a number, but they are not necessarily being taught these skills in the same way as their parents and grandparents were. Today, *understanding* mathematics is as much a classroom focus as finding the correct answer is.

Consider the following recent incident:

In a restaurant, a cashier attempted to add two bills, one for \$4.50 and one for \$5.50, by carefully lining up the decimals and "carrying," like this:

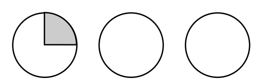
$$\begin{array}{r} {}^{1}5.50 \\ +4.50 \\ \hline 10.00 \end{array}$$

Although the procedure used was correct, the customer wondered why the cashier did not just add 4 + 5, see that 50 cents + 50 cents is another dollar, and know that the total was \$10. Certainly people should be able to perform this type of computation mentally, without a calculator or even a piece of paper. Many people cannot do so, however, partly because they lack number sense. The cashier learned the procedure in school but may not have learned enough about the nature of numbers to add them mentally.

A look back at history shows that difficulty with fractions and decimals has been with us for decades. For example, when eighth graders were asked in 1977 to estimate the sum of 12/13 + 7/8, only 10 percent of them selected the correct answer on a multiple-choice national examination. When those same students were asked to "add the fractions" 7/15 and 4/9, about 40 percent of them were able to find the common denominator and add the fractions. In other words, four out of ten students could *do* the computation, but only one out of ten could *think* about the fractions 12/13 and 7/8, realize that both were close to 1, and estimate their sum to be 2. The authors of a report on the performance of students on this national exam in 1977 wrote that "only about 40 percent of the 17-year-olds appear to have mastered basic fraction computation" (Carpenter et al. 1980). Such reports set the tone for changes in the way that mathematics should be taught. This year, the National Research Council released a report highlighting research trends that show that students in the United States can often perform computation but have difficulty applying basic skills to simple problems (NRC 2001).

Think, for example, about how you learned to divide fractions in school. To do a problem such as 3 ÷ 1/4, you were probably taught to invert and multiply, like this: 3/1 × 4/1 = 12/1 = 12. Although this answer is correct, did you understand why you were inverting or what the answer meant? Moreover, could you have created a word problem that would require using this division problem to solve it?

In the contemporary classroom, we might look at the division problem in the following manner: Think of each of the circles below as representing 1. The shaded slice in the first circle represents 1/4. The question is, How many one-fourths are in 3? We can see that twelve of the one-fourth pieces, four "slices" per circle, are needed to fill all three of the circles.



We might apply this situation in the real world, as in the following problem: Jason has 3 pounds of hamburger and wants to make patties that each weigh 1/4 of a pound. How many patties can he make? The answer is 12 patties. We could also think of the problem as asking how many quarters are needed to make \$3. Through such thinking, we not only know what the answer is but also have a picture of it in our minds and real-life applications that go with it. These connections allow us to move on to more difficult problems, then to create strategies for dealing with fractions. Eventually, students will be able to perform basic operations on fractions, and through using hands-on materials, visual aids, and real-life problems, they are more apt to understand why the operations work the way they do. Today's students will not struggle with fraction, decimal, and percent concepts; unlike the cashier, they will be very comfortable performing mental-math operations in these areas.

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Families ASK

Take-Home Page

Families often ask about "doing math":

The work my child does in class looks very different from the work I did as a child. What happened to the practice worksheets containing 15 or 20 problems to answer?

First of all, consider the definition of a problem. A problem is something you do not immediately know how to solve. The type of problems referred to above are "exercises" rather than problems. A real problem is connected to a meaningful context and gives students something connected to real life to think about and solve. It helps them know not only what mathematics they need and the steps for doing it but they learn when they need to do it and why it works.

The mathematics taught years ago largely focused on teaching students how to perform procedures. Students were required to memorize definitions, rules, and formulas, and often they applied them without any thought. Students were considered good at mathematics if they could "do math" quickly and without much thought. Today we know that for students to be able to use school mathematics outside the classroom, they must understand what they are doing. By *understanding*, we mean that they know not only how to perform some operation like long division but [also] why the procedure works and what the result means. This deeper understanding enables students to know when it is appropriate to use what they have learned.

To better understand this point, consider one middle school mathematics curriculum called the Connected Mathematics Project, which contains a unit titled "Filling and Wrapping" by authors Lappan, Fey, Fitzgerald, Friel, and Phillips. The unit enables students to explore and develop a deep understanding of measurement related to three-dimensional shapes. Rather than give students the formula for volume and a page of problems to compute the volume of a three-dimensional shape, students actually experiment with the formula to determine why it works and what the variables represent before they start plugging in numbers. The work that students do is not restricted to finding the volume of an object but requires them to explain their thinking, the reasonableness of their answer, and the rationale behind why it works. A deeper understanding of the formula increases the chances that students will not only remember the formula but [also] better know when and how to use it.

For example, in the unit above, students compare the volume of a cone with the volume of a cylinder (in the shape of a soup can). Students use cans of various sizes as cylinders and make a cone that fits the can so that the tip of the cone touches the bottom of the can. Students trim around the base of the cone so that it fits snugly inside the rim of the can (see **fig. 1**). The act of trimming around the lip of the can to create the base of the cylinder helps the students see that the cylinder and cone have this shape in common. To find the volume of both a cylinder and a cone, students need the area of the base; in this case, it is the same size circle for both.

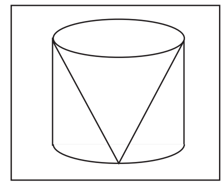

Fig. 1 Analyzing the volume of a cone versus a cylinder

Students then take the cone out of the can and fill it with small dried beans and empty it into the cylinder. They do this until the cylinder is completely filled. Students will find that they have to empty the cone into the cylinder three times to completely fill it. This helps students understand why the formula for the volume of a cone is 1/3 the volume of a cylinder. The formula for the volume of a cylinder is $\pi r^2 h$, whereas the formula for a cone is $1/3\pi r^2 h$. They will study the cone and cylinder and soon recognize that the area of a circle is part of the volume formula. The formula for the area of a circle is πr^2. They will then be able to describe the remaining part of the formula as being the height (h).

With a deeper understanding of volume formulas, students struggle less to determine which formula to use and where the numbers will be needed. In addition, they see the connections between the formulas, making the application of them more powerful. When students perform practice exercises, they can and should recognize and explain if their answers seem reasonable. If we change one dimension of the object, such as the height, students know that they do not have to start over to find the volume of the object. They recognize that they do not need to change part of their calculations but simply change the height in the formula to find the volume of the object.

With technological advances and societal changes, the type of mathematics that students need to do has changed. They need to be able to think, reason, and explain mathematics, which is reflected in how we teach students to "do math." It is much more than just numbers!

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Families ASK

Take-Home Page

Families often ask about calculator use:

In my child's classroom, calculators are always available. Shouldn't students have to know how to do calculations by hand first?

Consider the following reply:

Calculators and computers are essential tools for learning and doing mathematics. They compute efficiently and accurately and help us organize data and form mental pictures of mathematical ideas. The calculator should not become a replacement for basic understanding and skills. Instead, calculators should be used to help students develop knowledge and abilities for themselves. Middle school is an important time for students to extend their abilities to use technology in mathematics. Through experiences at school, students can learn when mental computation is the best strategy, when paper and pencil are practical, and when a calculator or computer is the tool of choice. Research has shown that students who use calculators appropriately in school mathematics do as well as, or better than, those who do not use calculators (Hembree and Dessart 1992).

You might also wonder whether children should use calculators for homework. Teachers design homework for different purposes at different times. For some assignments, the work is too complex and too time-consuming to complete without a calculator. Also, practice in computing by hand is not the goal of the activity. For other assignments, practice in computation is part of the goal, and the student is asked not to use a calculator. The teacher can tell students whether or not to use a calculator on any particular assignment.

At home, students can use calculators to explore problems they make up, such as "How many minutes old am I?" or "How much money would I have in a year if I earned a dollar an hour for three hours each day?" Students can keep track of costs at the grocery store and compare their totals with the receipt. They can plan car trips, calculating the cost of gasoline from the number of miles to be traveled and comparing their figures with the actual results.

Most middle school students have memorized the multiplication tables. Those who have not might enjoy practicing by using the constant function of the calculator. Calculators with a *constant function* allow the student to enter a number, such as 8, and continuously add that same number by pressing the symbols +, =, =, =,…, resulting in 8, 16, 24,…. Students can extend their multiplication facts to those less commonly used, such as 19, 38, 57,…. They can explore the powers of 2, which would result in 2, 4, 8, 16, 32, 64,…, or compare the circumference of the earth to the distance to the moon. They can also solve problems, such as the following:

> Atlanta's Hartsfield International Airport reported that in 1999 it had 78,092,940 passenger arrivals and departures. How many passengers traveled on a typical day? Per hour in 24 hours? (Beckmann and Golden 2001, p. 224) [Answers: About 214,000 per day; about 8910 per hour]

Calculators are the word processors of mathematics. A word processor cannot write a story by itself, and a calculator is useless unless it is in the hands of a person who has the problem-solving skills to know which buttons to push and when.

Tests, such as the SAT, ACT, PSAT/NMSQT, and the National Assessment of Educational Progress, are written to allow calculator use. Further, students can discover that mathematics helps them solve interesting problems when they use calculators as tools. Such activities can be fun and engaging and can motivate students to continue to take mathematics classes long after the minimum requirements have been met.

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From "Calculator in the Classroom" by Kay Gilliland in the November 2002 issue of Mathematics Teaching in the Middle School

Families ASK

Take-Home Page

Families often ask about using formulas:

Kids spend a lot of time building and measuring things in math class. Isn't it a lot quicker to just use a formula to solve problems—like I did and still do?

Teachers may answer the question this way:

Middle school students will learn formulas. Frequently, they will discover the formulas you had to memorize through *their* own investigations. Let's say, however, that a student simply memorizes these formulas about a cube (a box that is the same length in all three dimensions): (1) the volume of a cube is the length of a side cubed (side × side × side); and (2) the surface area of a cube is 6 times the area of the base (side length × side width × 6). So a cube with an edge of 4 inches has a volume of 64 (4 × 4 × 4) cubic inches and a surface area of 96 (16 × 6) square inches. The formula for the surface area of a cube is easy to remember: 6 times the area of the base, or 6 × length × width. However, without a variety of experiences with surface areas of different solids, a student may believe that this formula for the cube will work to find the surface area of all rectangular solids (such as any box, even if its length, width, and height are different measures). And the student will not recognize that this thinking is incorrect.

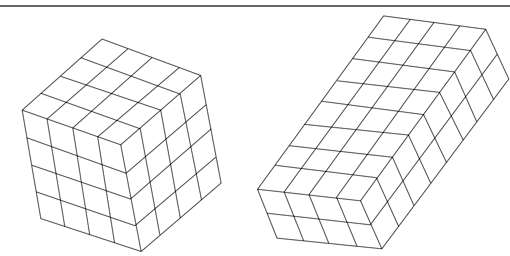

Fig. 1 Two different rectangular solids with a volume equal to 64 cubic units

However, if a student has measured many sizes of boxes, some cubes (fig. 1a), some rectangular prisms (solids) (fig. 1b), and compared their volumes with their surface areas, that student has the experience to recognize when an answer doesn't make sense. And, by looking at a graph of the investigation carefully, students can discover that they are not limited to boxes measured in whole numbers; they can estimate from the graph for the relationships among boxes with fractional numbers for sides.

A typical problem in a middle school textbook on this topic might be worded like this:

> A manufacturer wants to package 1-inch cubes as classroom manipulatives. The company wants to design an inexpensive box in the shape of a rectangular prism that exactly fits the 64 cubes. Find all the ways that 64 cubes can be arranged into a rectangular prism. Which arrangement would require the least amount of material for creating the box?

In solving this problem, students will need to create a chart with length, width, height, and volume and possibly a sketch of the resulting rectangular prism. They will need to consider only whole numbers because the cubes being packaged are a fixed size: 1-inch cubes.

Volume in Cubic Inches	Length in Inches	Width in Inches	Height in Inches	Surface Area in Square Inches
64	64	1	1	256
64	32	2	1	196
64	16	4	1	168
64	8	8	1	152
64	16	2	2	136
64	8	4	2	112
64	4	4	4	96

In such activities, students learn how to compare volume with surface area and how various size measurements influence each other. They observe patterns in the tables and graphs that they construct. They notice that if they keep the volume the same, long, thin prisms (boxes) are less cubelike and have larger surface areas than chunkier boxes that are more cubelike. These experiences help students to understand the basic ideas behind the formulas, so they remember where and when the formulas will apply. You can see that the mathematics we are teaching middle school students goes well beyond memorizing formulas. Instead, it is all about *understanding* the formulas and *using* them to analyze problem-solving situations.

From "Why Not Just Use a Formula?" by Kay Gilliland in the May 2002 issue of **Mathematics Teaching in the Middle School**

Families ASK

Take-Home Page

Families often ask about timed tests:

My child never does timed tests in math class. I thought that speed and getting right answers quickly were important in mathematics.

Consider the following reply:

Middle school is a time for children to broaden their understanding of mathematics and to develop persistence and flexibility in their approach to mathematics problems. Adult mathematicians often spend weeks, months, or even years on a single problem. Students need to learn the value of continuing to work on a problem until they finally come up with answers that appear to be reasonable, correct, and useful. They need to have experience in defending their answers, justifying the steps they took, and communicating their findings clearly. All of this is a matter not of speed but of dedication. In *About Teaching Mathematics,* Marilyn Burns writes the following:

> What about using timed tests to help children learn their basic facts? This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning.
>
> Also, timed tests do not measure children's understanding. . . . It doesn't ensure that students will be able to use the facts in problem-solving situations. Furthermore, it conveys to children that memorizing is the way to mathematical power, rather than learning to think and reason to figure out answers. (2000, p. 157)

Most middle school students have memorized the hundred or so most useful single-digit addition, subtraction, multiplication, and division facts. If not, this task can be accomplished without putting students through the anxiety of timed tests. Of course, if a child dearly loves to do timed tests and glories in beating his or her previous times, this endeavor can be enjoyed at home in the way that any other race or game is enjoyed, but it is not essential to doing mathematics.

At the heart of mathematics teaching is the view that students learn when they encounter problems in context; act on physical objects; use appropriate tools; and talk about, and reflect on, mathematical ideas. This view is described in *Everybody Counts* by the National Research Council (1989):

> Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. . . . This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning. (pp. 58–59)

Students need to know the single-digit addition, subtraction, multiplication, and division facts, but timed tests are not necessary and often do more harm than good. The study of mathematics goes well beyond getting simple answers quickly.

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Families ASK

Take-Home Page

Families often ask about measurement:

What are children learning about measurement in today's classrooms?

Measurement skills are frequently used in the real world. Measurement means many things, from measuring the size of a room to measuring the time it takes to run a mile to the amount of water used when a faucet drips. The regular occurrence of measurement opportunities would make it reasonable to assume that students learn measurement concepts and skills easily, which is not necessarily the case. When looking at how well students learn measurement, the gap between what students are taught and what they can actually do is large. You might be wondering why, and what can be done about it.

This gap occurs when the way that students are taught does not match how they perform with the information. For example, when students are given a page with pictures of objects and each object has a picture of a ruler aligned properly under it, students simply practice reading the ruler to measure the object. This scenario is not how measurement is performed in the real world and results in the inability not only to perform measurement procedures but also to understand what the concept of measurement really means.

Realistically, students will need to measure an object for a specific purpose. They will have to determine for themselves what needs to be measured. Do they need to measure the length of the object? Or is it necessary to find the area, which is what it takes to "cover" a surface? Maybe the problem they have encountered involves measuring volume, which is how much it takes to fill a three-dimensional space. The weight or temperature of the object may be the feature in question. Regardless, the student must identify what needs to be measured and select the appropriate tool. The tool might be a ruler, tape measure, measuring cup, scale, thermometer, or some other device. That is why the best way to teach measurement in school is to find or create situations in which students need to measure and let them experience this process. It enables them to see measurement as a hands-on experience that involves many decisions.

Students learn in school that measurement is connected to many other things. Students learn fractions as they measure, to a fraction of an inch, objects that are less than one foot. They see connections to geometry when they study shapes and consider the measurements of the perimeter and area of those shapes. Not only do students experience measurement in the study of other mathematics concepts but also in the study of other subjects, such as social studies, science, and art.

We can also close the gap in school versus real-life measurement skills by enabling students to learn measurement formulas in a meaningful way. Students can learn the formulas for the surface area of a three-dimensional shape like a box by carefully "covering" the surface of the shape with grid paper, then removing the paper to study the area. Using boxes of various sizes enables students to see a pattern in determining surface area. They can then generalize this pattern into a formula to use in the future and to apply to other three-dimensional shapes, as well.

At home, you can provide your child with opportunities to measure. When you are cooking, ask your child to get you one-fourth cup of sugar. [She or he] will need to decide what tool is needed to measure the ingredients. If you are grocery shopping, ask your child to get a certain dollar amount of a fruit or vegetable that is paid for by the pound. [He or she] will need to recognize that a weight measurement is necessary and will have to find the scale to measure out the amount needed. Point out labels on bottles and containers; they contain "capacity" or volume measurement information that tells how much the bottle or container can hold. You can talk about objects, their measurable characteristics, and what tools could be used to measure them.

The best way to teach students about measurement is to give them opportunities to measure, then discuss options with them. Measurement is frequently encountered in the real world, so the goal is for students to determine what to measure, decide on an appropriate tool, select a reasonable unit of measure, and check the reasonableness of their results.

From "Measurement: What's the Big Idea?" by Sherri Martinie in the April 2004 issue of **Mathematics Teaching in the Middle School**