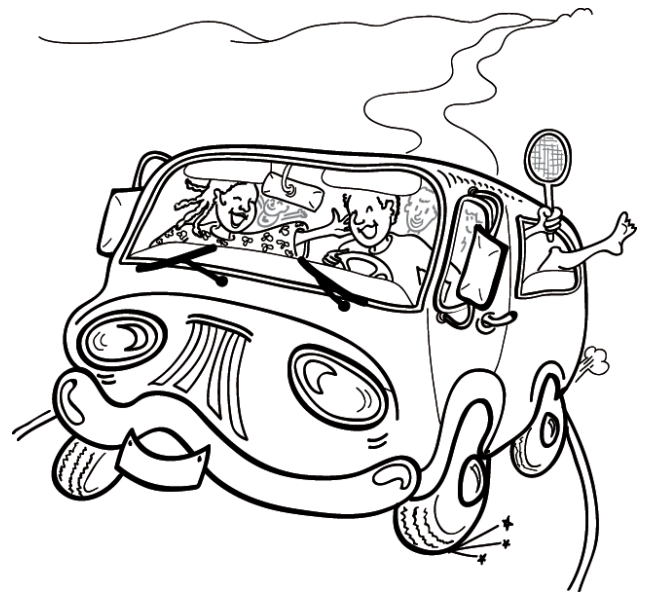




*This brainteaser was written by Derrick Niederman.*

When Julie's family travels, her father always drives, and her mother always sits in the front passenger seat. Julie and her siblings sit in the middle and back row of their van.

Julie told her brothers and sisters, "Of all the ways that two of us can sit in the middle row, I'm involved in one-third of those pairs."



How many siblings does Julie have?



**Solution: 5 siblings.**

You can solve this puzzle by “acting it out.” If Julie had one sibling, Alison, there would only be one way for two of them to sit in the middle row, and Julie is involved in that pair. The ratio is  $1/1$ , which is more than the  $1/3$  required.

If Julie had two siblings, Alison and Barack, there would be three ways to form a pair: JA, JB, or AB. Julie is involved in two of those pairs. The ratio is  $2/3$ , still higher than required.

Similar reasoning could be used for three or four siblings. With three siblings, Julie is involved in three of six pairs; with four siblings, she is involved in four of ten pairs—still not quite there.

With five siblings, things finally work out. If her siblings are Alison, Barack, Camryn, David, and Edana, then the following pairs are possible:

JA	AB	BC	CD	DE
JB	AC	BD	CE	
JC	AD	BE		
JD	AE			
JE				

There are 15 pairs total, and Julie is involved in five of them (those in the first column). This gives the ratio  $5/15 = 1/3$ , as required.

A more algebraic solution takes note of two facts. First, if Julie has  $n$  siblings, then  $n$  pairs of siblings include her: she forms a pair with each of her siblings. Second, the total number of pairs is  $1/2(n)(n + 1)$ , or the  $n$ th triangular number. Therefore, the ratio of the number of pairs in which Julie is involved to the total number of pairs is given by the expression below, which is only equal to  $1/3$  when  $n = 3$ .

$$\frac{n}{\frac{1}{2}(n)(n+1)} = \frac{2n}{n(n+1)}$$

Finally, a geometric solution is shown by representing Julie and each of her siblings as a vertex of a polygon, and pairs of siblings are represented by diagonals. The pairs in which Julie is involved are shown by black lines, and all other pairs are shown by light gray lines. In a hexagon, there are five diagonals that have Julie as an endpoint, and there are 15 diagonals total, so Julie is involved in  $5/15 = 1/3$  of the pairs.

