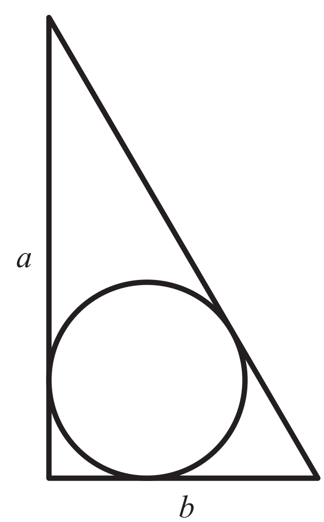
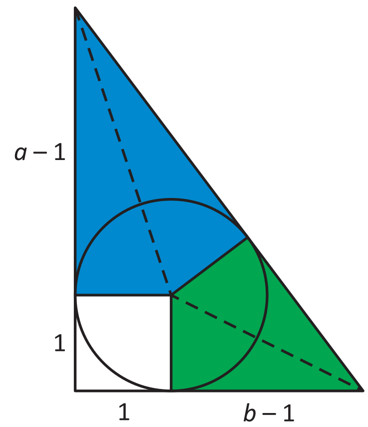
This brainteaser was written by Derrick Niederman.

A circle of radius 1 unit is inscribed inside a right triangle that has height *a* and base *b*. If *b* is an integer, what are the possible values of *a*?



Solution: 4, 3, **8/3.**

The first thing to note is that the diameter of the circle is 2 units. Since the height of the triangle must be greater than the diameter of the circle, then $a > 2$. (Because we cannot logically distinguish between a and b due to symmetry, then $b > 2$, as well.)



Draw three radii, each perpendicular to a side of the triangle. This divides the triangle into three regions: a blue kite in the upper left, a green kite on the bottom right, and a square of side length 1. The blue kite consists of two triangles, each with height a – 1 and base 1, so it has area $a - 1$. Similarly, the green kite consists of two triangles with height $b - 1$ and base 1, so it has area $b - 1$. And the square, obviously, has area 1. Adding these pieces gives the following expression for the area of the triangle: $a + b - 1$.

However, we also know that a triangle with height a and base b has an area of ½ab. These two different expressions for the area lead to the following equation:

$$\tfrac{1}{2}ab = a + b - 1$$
$$ab = 2a + 2b - 2$$
$$ab - 2a = 2b - 2$$
$$a = \frac{2b-2}{b-2}$$

We can now test this equation for integer values of $b > 2$, which lead to the following:

b	a
3	4
4	3
5	$\frac{8}{3}$
6	2

At this point, we can stop. Remember that $a > 2$, so although there are values of $a < 2$ that satisfy the equation, they are not reasonable in the context of the problem. Therefore, we needn't test any values of $b > 6$, and the three possible values of a are 4, 3, and $\frac{8}{3}$.