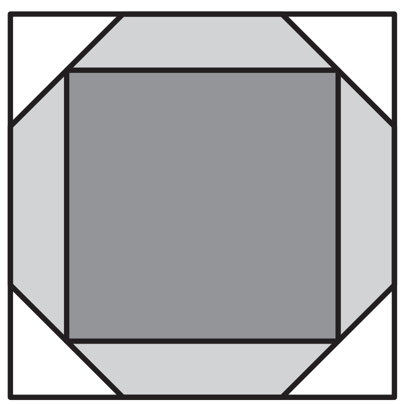
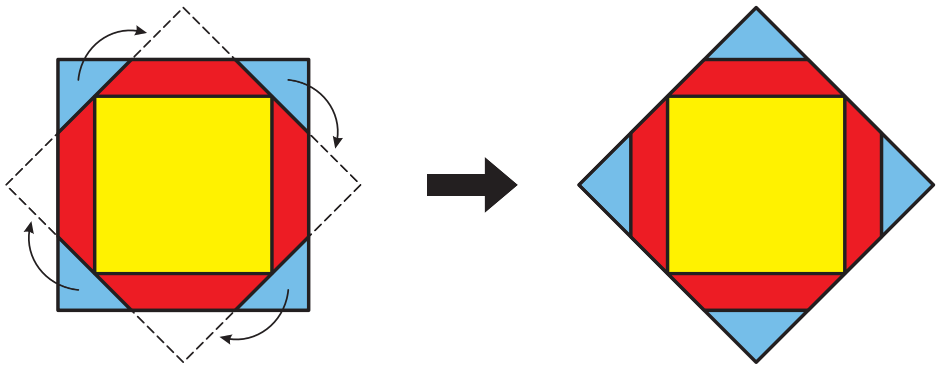
This brainteaser was written by Derrick Niederman.

A regular octagon is inscribed inside a square. Another square is inscribed inside the octagon. What is the ratio of the area of the smaller square to the area of the larger square?

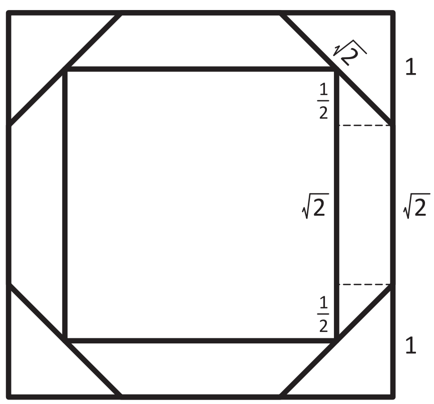


Solution: ½.

One way to see this is to rotate the blue triangles onto the top of the red trapezoids, with the hypotenuse of the triangle flush with the shorter base of the trapezoid. Then it's pretty easy to see that if the four red and blue triangles are folded over, they'd completely cover the yellow square. In other words, the area of the yellow square is equal to the area of red trapezoids and blue triangles combined, so the ratio of the smaller square to the larger square is ½.



It is also possible to calculate the area of the larger and smaller squares. Start by assuming that the length of the shorter sides of the triangles is 1 unit, as shown below. Then the hypotenuse of each triangle is $\sqrt{2}$, and since the hypotenuse is also a side of the regular octagon, then all sides of the octagon are $\sqrt{2}$.



Consequently, the side length of the larger square is $2+\sqrt{2}$, and the side length of the smaller square is $1+\sqrt{2}$, so their respective areas are $\left(2+\sqrt{2}\right)^2 = 6+4\sqrt{2}$ and $\left(1+\sqrt{2}\right)^2 = 3+2\sqrt{2}$. Dividing the area of the smaller square by that of the larger yields ½.

If those calculations are a little too messy for you, then here is an alternative solution. Note that the length of the diagonal of the smaller square equals the distance between opposite sides of the regular octagon. However, the side length of the larger square is also equal to the distance between opposite sides of the octagon. Therefore, the side length of the larger square is $\sqrt{2}$ times the side length of the smaller square, so the ratio of the areas is ½.