This brainteaser was written by Derrick Niederman.

A pocket watch is placed next to a digital clock. Several times a day, the product of the hours and minutes on the digital clock is equal to the number of degrees between the hands of the watch. (The watch does not have a second hand.) As you can see, 10:27 is not one of those times — the angle between the hands is not 270°. If fractional minutes aren't allowed, find the times at which the product of the hours and minutes is equal to the number of degrees between the hands.

Solution: 3:36 and 11:20.

The angle between the hour and minute hands equals the product of the hours and minutes at 3:36 and 11.20. In the latter case, the product equals 220, which is the number of degrees between the hands—if you go the long way around!

Let h = hours and m = minutes. The hour hand covers 30° every hour, so at the top of any hour it has traveled 30h degrees, and with every passing minute it travels another m/2 degrees. Given that the minute hand moves 6° every minute, the number of degrees between the hands is given by either $6m - (30h + m/2)$ or $(30h + m/2) - 6m$. Consequently, the relevant equations are:

$$hm = 6m - \left(30h + \tfrac{m}{2}\right) \quad \rightarrow \quad m = \frac{60h}{11 - 2h},$$

$$hm = \left(30h + \tfrac{m}{2}\right) - 6m \quad \rightarrow \quad m = \frac{60h}{2h + 11},$$

Because m is a positive integer, the first equation implies that $11 - 2h$ must divide evenly into $60h$, and that happens only when $h = 3$. Plugging in $h = 3$ yields $m = 36$, so (3, 36), or 3:36, must have the desired property. Sure enough, there are precisely 3 × 36 = 108° between the hour and minute hands at 3:36.

For the second equation, the right-hand side must be a positive integer, and that happens only when $h = 11$. In that instance $m = 20$, producing (11, 20), or 11:20, as our second time. The fact that you go the "long way around" in this instance arises because the second equation essentially reverses the order of the hour and minute hands.