This brainteaser was written by Derrick Niederman.

The number 4 can be expressed as the sum of three positive integers in only one way:

$$4 = 1 + 1 + 2$$

However, the number 50 can be expressed as the sum of three positive integers in 200 ways.

Somewhere in between, there is a number *n* that can be expressed as the sum of three positive integers in precisely *n* ways. Can you find *n*?

Solution: 12.

A *partition* is a way of writing an integer as a sum of smaller integers. For instance, 4 + 5 + 6 is a partition of 15, and 11 + 10 + 9 + 7 + 2 is a partition of 39. This problem is concerned with partitions that contain just three addends.

There is only one partition of 3 that contains three addends, namely, 1 + 1 + 1. The trick to this puzzle is realizing that, as the integer increases, the number of partitions either increases or stays the same. For example, there is only one partition of 4 with three addends, 1 + 1 + 2, so the number of partitions stayed the same as the integer increased from 3 to 4. However, there are two partitions of 5 that contain three addends, 1 + 2 + 2 and 1 + 3 + 1, so there are more partitions for 5 than there are for 4.

This is helpful, because it means we can limit the number of integers we have to check. Knowing that 3 has only one partition and that 50 has 200, we would suspect that the number we are looking for would be closer to 3 than to 50. So, take an educated guess and try 15. As it turns out, there are 19 partitions of 15 that contain three addends (you can create a list to prove this to yourself). That's too high, since 19 > 15. Searching lower, we might then check 10, which we'd find has just 7 partitions. That's too low, since 7 < 10.

This now significantly limits the range we have to check, and a little more fiddling will lead to the solution. Below are the twelve partitions of 12 with three addends:

1 + 1 + 10	1 + 2 + 9	1 + 3 + 8	1 + 4 + 7
1 + 5 + 6	2 + 2 + 8	2 + 3 + 7	2 + 4 + 6
2 + 5 + 5	3 + 3 + 6	3 + 4 + 5	4 + 4 + 4