



*This brainteaser was written by Derrick Niederman.*

If  $x^2 + y^2 = 36$ ,  $xy = 32$ , what is  
the positive value of  $x + y$ ?



**Solution: 10.**

The first equation,  $x^2 + y^2 = 36$ , represents a circle of radius 6 with its center at the origin. The second equation,  $xy = 32$ , represents a hyperbola that never intersects the circle. Consequently, there are no real solutions to this system of equations. However, there are complex roots, and carrying out the entire calculation using complex numbers would lead to the correct result. Of course, that's messy and difficult, so let's look for a better method.

We have a value for  $x^2 + y^2$ , we have a value for  $xy$ , and we want to know the value of  $x + y$ . Is there some equation from algebra that relates these three pieces? Indeed, there is—the square of a binomial! That is,

$$\begin{aligned}(x + y)^2 &= x^2 + 2xy + y^2 \\ &= x^2 + y^2 + 2xy\end{aligned}$$

Consequently, by substituting the values we know, we get  $(x + y)^2 = 36 + 2(32) = 100$ . Therefore,  $x + y = 10$ .