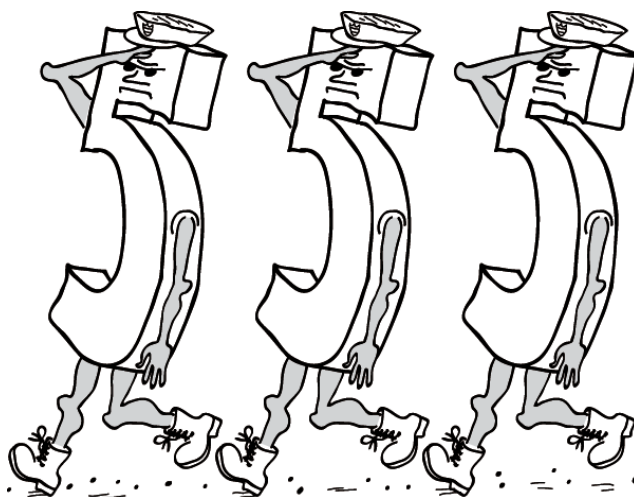


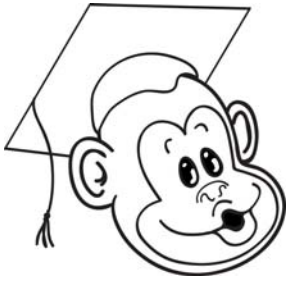


*This brainteaser was written by Derrick Niederman.*

What is the smallest positive integer that, when multiplied by 7, yields a product in which all of the digits are 5's?



(Can you find the answer without a calculator?)



**Solution: 79,365.**

One way to tackle the problem is to find the first number consisting of all 1's (called a *repunit*) that is divisible by 7. To do that, just set up a standard long division, adding as many 1's as you need until you get a remainder of zero. You'll get the equation  $111,111 = 7 \times 15,873$ , and because this equation is the smallest of its kind, we can multiply both sides by 5 to yield  $555,555 = 7 \times 79,365$ .

You can also multiply backwards to find the answer. That is, what number times 7 gives a 5 as the units digit in the result? Obviously, that's 5. But then  $5 \times 7 = 35$ , so a 3 carries to the tens column. So then, what number times 7, and then adding the carried 3, gives 5? That's 6, since  $6 \times 7 + 3 = 45$ . That gives a 4 that carries... and so on. This process will eventually lead to the answer, too.

Of course, the method that requires the least amount of thinking is to just use a calculator. Enter a string of 5's, divide by 7, and keep increasing the number of 5's until the answer is an integer.