This brainteaser was written by Derrick Niederman.

Write the number 41 in a box. Now move in a counterclockwise direction, creating new boxes and each time adding 1 to the number inside. This spiral starts out as follows:

45	44	**43**	
46	**41**	42	51
47	48	49	50

Note that the numbers in bold — 41, 43, and 47 — are prime numbers (numbers whose only divisors are themselves and 1), and they occur along a diagonal. If you keep filling in the spiral, what is the first number that is not prime to appear along this diagonal?

Solution: 1681. Which, as it turns out, is equal to 41^2.

This problem involves a diagram, so you might be wondering why a diagram isn't included with this solution. It's because the diagonal prime sequence gets ridiculously long, and the diagram would need to be very large. But there is a logical way of getting at the solution.

The sequence of numbers along the diagonal is 41, 43, 47, 53, 61, 71, 83, …, and the differences between consecutive numbers in this sequence are 2, 4, 6, 8, 10, 12, …. The pattern continues in this manner, each time adding two more than was added previously. The list below shows all the numbers along the diagonal up to 1681, the first number in the list that is not prime.

97	223	421	691	1033	1447
113	251	461	743	1097	1523
131	281	503	797	1163	1601
151	313	547	853	1231	1681
173	347	593	911	1301	
197	383	641	971	1373	

Proving that all the numbers in the list less than 1681 are prime requires showing that each number has no factors other than 1 and itself. In other words, we have to show that no integer less than the square root of the number divides evenly into the number. Doing this requires a lot of calculation, but a spreadsheet can be used to perform the calculations.

Alternatively, if you remember that the differences between successive square numbers, starting with 1, are 3, 5, 7, 9, and so on, it is not surprising that the diagonal elements of our sequence can also be described using squares — that is, with a quadratic expression. Specifically, the values along the diagonal are the values of the expression $x^2 - x + 41$, for integer values of x. This expression goes all the way back to Leonhard Euler and has the remarkable property that it produces primes for the first 40 positive integers. Obviously, the expression yields a composite number when $x = 41$, because each of the individual terms is divisible by 41. So $41^2 = 1681$ is the answer we are looking for.