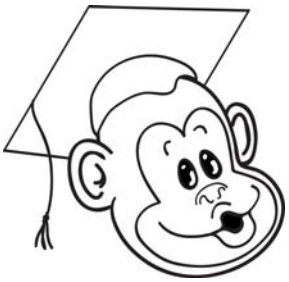
This brainteaser was supplied by the Mathematical Olympiads for Elementary and Middle Schools (www.moems.org).

The five-digit number $3a,5a7$ is a multiple of 9, and each a represents the same digit. What digit does a represent?



Solution: 6.

Because there are only ten possible values for *a*, namely 0–9, one possible solution strategy is to try each of the digits and see which one gives a result that is evenly divisible by 9. That will work, but there's a more sophisticated approach.

A number is evenly divisible by 9 if (and only it) the sum of the digits is a multiple of 9. For instance, the number 297 is divisible by 9, and the sum of its digits is 2 + 9 + 7 = 18, which is a multiple of 9. On the other hand, the sum of the digits of 263 is 2 + 6 + 3 = 11, and since 11 is not a multiple of 9, then 263 is not evenly divisible by 9.

For this problem, then, it must be the case that 3 + *a* + 5 + *a* + 7 = 15 + 2*a* is a multiple of 9. The value of 15 + 2a is a multiple of 9 when *a* = 1.5, 6, 10.5, 15, 19.5, 24, …, but the only number in this list that is an integer from 0 to 9 is 6.

As a check, note that 36,567 ÷ 9 = 4,063, with no remainder.