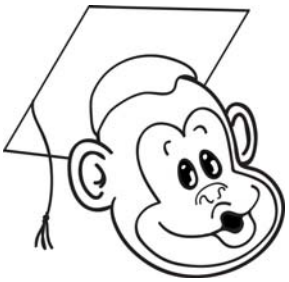
This brainteaser was written by Derrick Niederman.

There are 4! = 24 ways to rank four objects. However, a friend told me that if ties are allowed, the number increases to 75.

I attempted to list all the possibilities by first listing the 24 orderings of four objects, then using brackets to group ties involving two players, then group ties involving three players, and finally the single case in which all four objects are tied. But something has gone wrong; my list includes just 69 possibilities, not 75.

ABCD	ABDC	ACBD	ACDB	ADBC	ADCB	24
BACD	BADC	BCAD	BCDA	BDAC	BDCA	
CABD	CADB	CBAD	CBDA	CDAB	CDBA	
DABC	DACB	DBAC	DBCA	DCAB	DCBA	
[AB]CD	[AB]DC	[AC]BD	[AC]DB	[AD]BC	[AD]CB	12
[BC]AD	[BC]DA	[BD]AC	[BD]CA	[CD]AB	[CD]BA	
A[BC]D	A[BD]C	A[CD]B	B[AC]D	B[AD]C	B[CD]A	12
C[AB]D	C[AD]B	C[BD]A	D[AB]C	D[AC]B	D[BC]A	
AB[CD]	AC[BD]	AD[BC]	BA[CD]	BC[AD]	BD[AC]	12
CA[BD]	CB[AD]	CD[AB]	DA[BC]	DB[AC]	DC[AB]	
[ABC]D	[ABD]C	[ACD]B	[DAB]C			4
A[BCD]	B[ACD]	C[ABD]	D[ABC]			4
[ABCD]						1
					Total	69

What happened? Did I miss something, or was my friend mistaken?



Solution: I missed 6 possibilities.

My friend was correct. When four objects are ranked, there are 75 possibilities when ties are included.

Missing from the list are the six cases in which *two* pairs are tied. Using the notation of the original problem, here they are:

[AB][CD] [AC][BD] [AD][BC] [CD][AB] [BD][AC] [BC][AD]