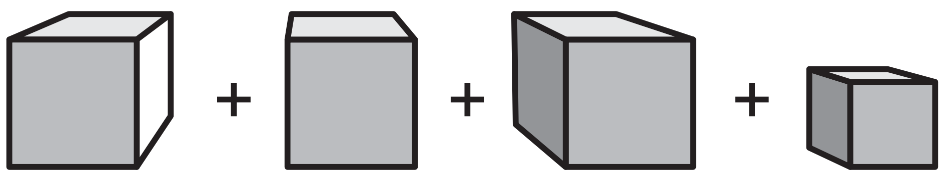
This brainteaser was written by Derrick Niederman.

According to Waring's theorem, any positive integer can be represented as the sum of nine or fewer perfect cubes (not necessarily distinct).



For instance, 89 can be represented as the sum of four perfect cubes: 27 + 27 + 27 + 8 = 89.

Can you express 239 as a sum of nine or fewer perfect cubes?

Solution: 64 + 64 + 27 + 27 + 27 + 27 + 1 + 1 + 1
125 + 27 + 27 + 27 + 8 + 8 + 8 + 8 + 1

As you may have guessed, the number 239 wasn't chosen at random. It has the unusual property of being the largest number that cannot be represented with *fewer* than nine cubes (23 is the only other number requiring nine).

The only numbers that can be used to add to 239 are 1, 8, 27, 64, 125, and 216 — the cubes of the first six integers, respectively, each of which is less than 239.

The representations we're looking for start with 64 or 125 and go like this:

$$239 = 64 + 64 + 27 + 27 + 27 + 27 + 1 + 1 + 1$$
$$= 4^3 + 4^3 + 3^3 + 3^3 + 3^3 + 3^3 + 1^3 + 1^3 + 1^3$$
$$239 = 125 + 27 + 27 + 27 + 8 + 8 + 8 + 8 + 1$$
$$= 5^3 + 3^3 + 3^3 + 3^3 + 2^3 + 2^3 + 2^3 + 2^3 + 1^3$$