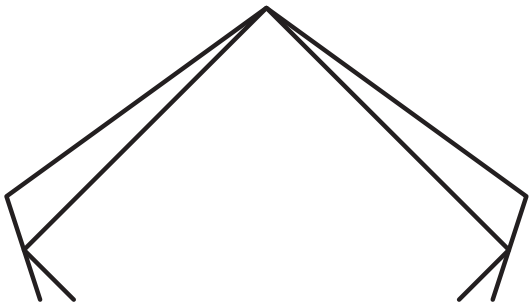
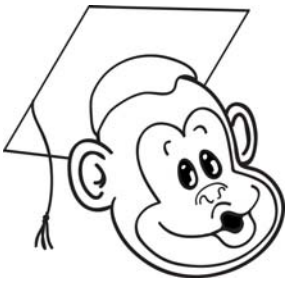
This brainteaser was written by Derrick Niederman.

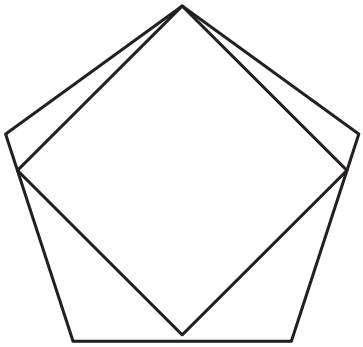
The diagram below shows the top of a regular pentagon with the top of a square inscribed in it. The shapes share a vertex at the top, and the other two vertices of the square lie on the sides of the pentagon. If the diagram were continued to include the entire pentagon and the entire square, which shape would extend below the other?



In other words, does the whole square fit inside the pentagon, does the square protrude at the bottom, or do the square and pentagon meet at a single point?



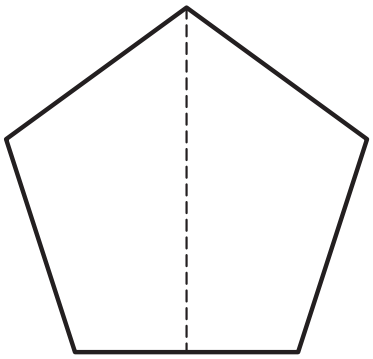
Solution: The square fits inside the pentagon.



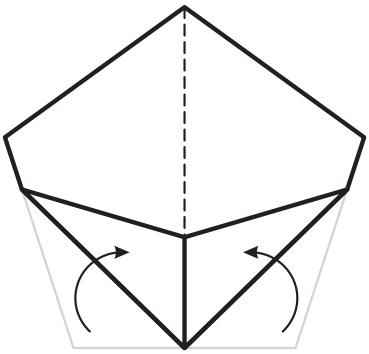
If you are very careful and draw the square perfectly inside the pentagon, you'll see that the fourth vertex of the square lies within the pentagon.

To just see the picture, however, is quite unsatisfying. After all, if your drawing is off by just a little bit, it could appear that the fourth vertex lies on the bottom side or even outside the pentagon. How can you know for sure? You could use trigonometry, but that gets very ugly. Isn't there a more elegant approach?

Perhaps the visual "proof" offered by the following series of folds might convince you.

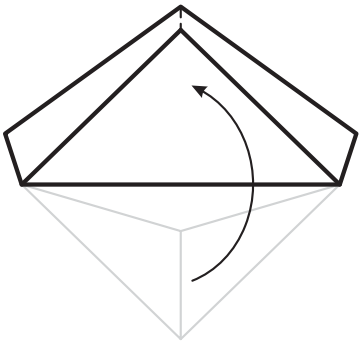


First, fold the pentagon vertically along the midline.



Next, fold the bottom corners of the pentagon so that the base lies on the midline. Doing this will form a right angle at the center of the base.

Why do this? Well, if the fourth vertex of the square lies exactly on the base of the pentagon, then a right angle has to touch the midpoint of the base (which is what was just created). Further, the points at which these two newly formed triangles touch the sides of the pentagon are the points where the vertices of the square must lie.



Finally, fold the bottom portion of the paper so that a horizontal fold connects the points where the triangles from the previous fold meet the sides of the pentagon.

As can be seen, the bottom vertex does not reach the top of the pentagon. This indicates that a square inscribed in the pentagon must have a diagonal that is shorter than the height of the pentagon. Consequently, an inscribed square will lie entirely within the pentagon.