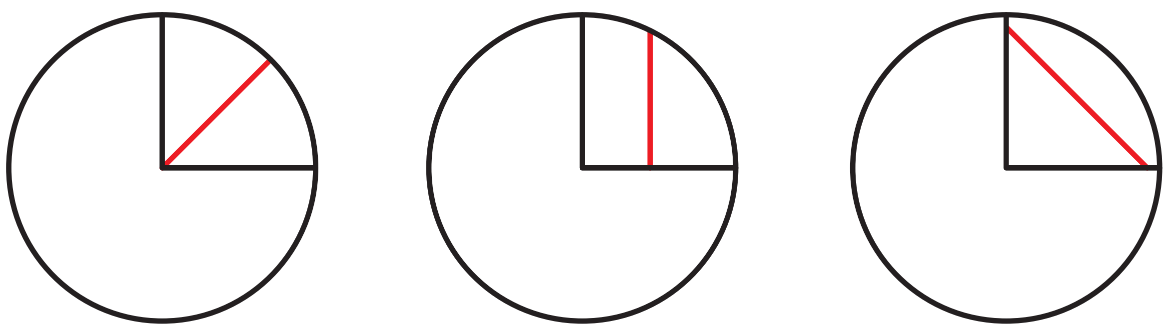
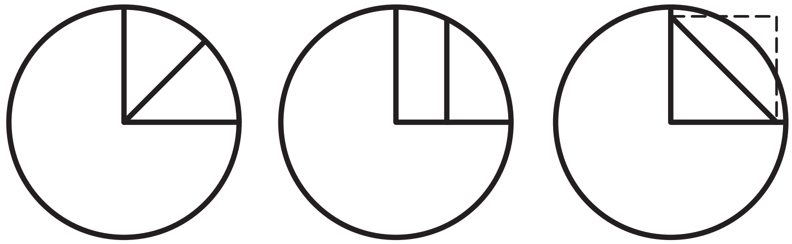
This brainteaser was written by Derrick Niederman.

In the diagram below, three different line segments each divide a quarter-circle into two regions of equal area. Rank those three segments from shortest to longest.



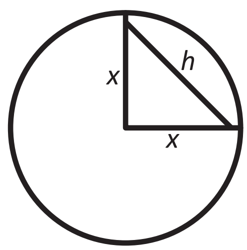
Solution: middle, shortest, longest.

The length of the segment on the left is simply the radius of the circle, and the segment in the second circle is clearly shorter than the radius.



So the question is, is the segment in the third circle longer or shorter than the other two? If you think of the segment in the third circle as the diagonal of a square, that square must extend beyond the circle for the segment to bisect the area. Consequently, the diagonal must be greater than the radius of the circle, so the third segment is the longest.

But perhaps that picture doesn't convince you. A more rigorous proof is to express the length of the segment in terms of the radius. Let x represent the lengths of the legs of the triangle, and h is the length of the segment in question.



The area of the triangle is $A = \frac{1}{2}x^2$. But its area is also $A = \frac{1}{2}(\frac{1}{4}\pi r^2)$, because the triangle is one of two equal halves in the quarter-circle. This leads to an equation that can be solved for x in terms of r.

$$\tfrac{1}{2}x^2 = \tfrac{1}{2}\left(\tfrac{1}{4}\pi r^2\right)$$
$$x^2 = \tfrac{1}{4}\pi r^2$$
$$x = r\sqrt{\tfrac{\pi}{4}}$$

The hypotenuse of the triangle, h, is the segment about which we are concerned. Its length can now be found with the Pythagorean theorem.

$$h^2 = x^2 + x^2 = 2x^2$$
$$h^2 = 2\left(r\sqrt{\tfrac{\pi}{4}}\right)^2$$
$$h = \tfrac{\pi}{2}r \approx 1.57r$$

Because $h > r$, the third segment is greater than the radius, so it is the longest.