



This brainteaser was written by Derrick Niederman.

The *factorial* of n is the product of all positive integers less than or equal to n . It is represented as $n!$. An example with $n = 8$ is shown below. With that in mind, can you find three sets of numbers (a, b, c) such that $a! \times b! = c!$ and $a < b < c < 25$?

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$



Solution: (3, 5, 6), (4, 23, 24), and (6, 7, 10).

Two combinations are relatively easy to find: $3! \times 5! = 6!$ and $4! \times 23! = 24!$. These rely on the facts that $3! = 6$ and $4! = 24$. In general, $n!(n! - 1)! = (n!)!$.

The third one is a little harder to find: $6! \times 7! = 10!$. When we multiply things out for $6!$, we get the following result:

$$\begin{aligned}6! &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= (2 \times 3) \times 5 \times 4 \times 3 \times 2 \\ &= 2 \times 4 \times 3 \times 3 \times 2 \times 5 \\ &= 8 \times 9 \times 10\end{aligned}$$

So $7! \times 6!$ is equivalent to $7! \times 8 \times 9 \times 10$, which is equal to $10!$.