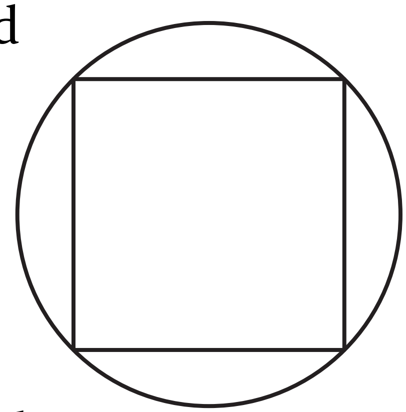
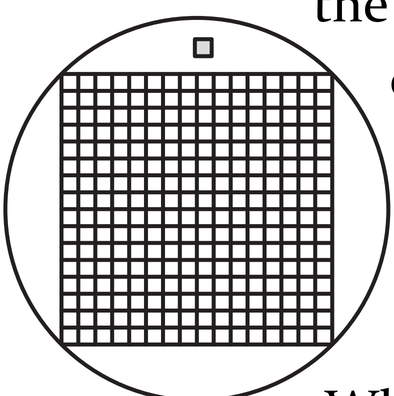
This brainteaser was written by Derrick Niederman.



To the right is a circle with an inscribed square. Obviously, there isn't room for another nonoverlapping square of the same size within the circle.

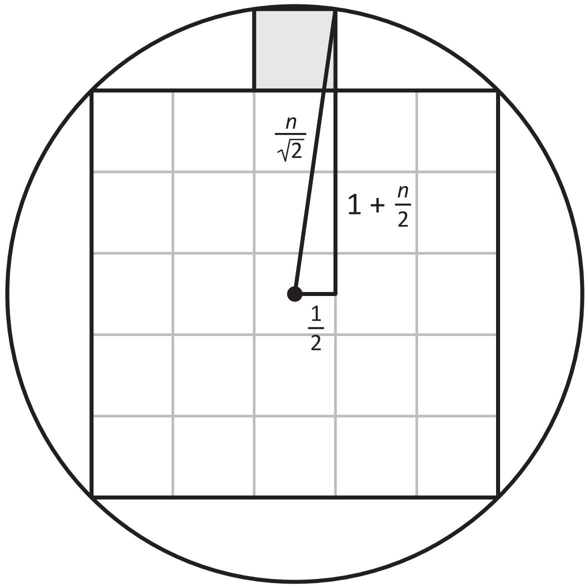
But suppose that you divided the square into n^2 smaller squares, each with side length $1/n$. Would one of those smaller squares fit in the space between the large square and the circle? As shown to the left, this works if $n = 16$ and the large square were divided into 256 smaller squares. But it would work for smaller values of n, too.



What is the smallest value of n such that one of the smaller squares would fit between the larger square and the circle?

Solution: *n* = 5.

If the square is divided into n^2 smaller squares, each of which is 1 × 1, then the larger square is $n \times n$. Take one of the smaller squares and center it in the space between the circle and the large square, as in the figure below. Then, if the center of the circle is connected to the upper right vertex of the added square, a right triangle is formed with dimensions shown.



The Pythagorean theorem can then be applied, and the results can be simplified as follows:

$$\left(\tfrac{1}{2}\right)^2 + \left(1 + \tfrac{n}{2}\right)^2 = \left(\tfrac{n}{\sqrt{2}}\right)^2$$
$$\tfrac{1}{4} + 1 + n + \tfrac{n^2}{4} = \tfrac{n^2}{2}$$
$$1 + 4 + 4n + n^2 = 2n^2$$
$$n^2 - 4n - 5 = 0$$
$$(n-5)(n+1) = 0$$

The solutions to this equation are *n* = 5 and *n* = -1, the latter of which does not make sense for this situation. Consequently, the square will fit if *n* has any value greater than or equal to 5.