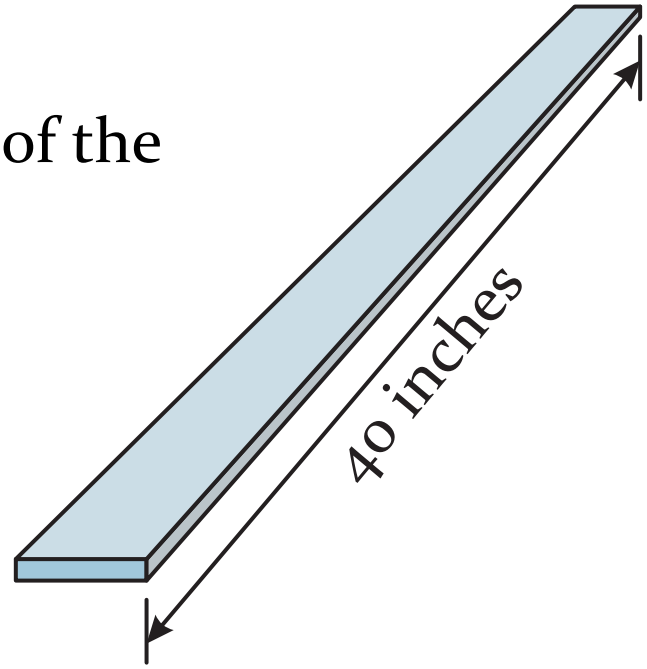
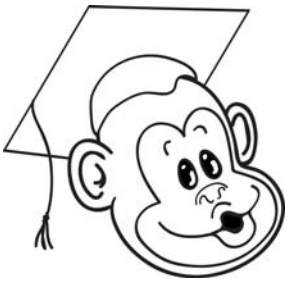
This brainteaser was written by Patrick Vennebush.

A 40-inch straightedge (without markings) is divided into four pieces. The length of each piece is an integer number of inches. These four pieces, when used in tandem, can be used to measure any integer length from 1 to 40 inches.

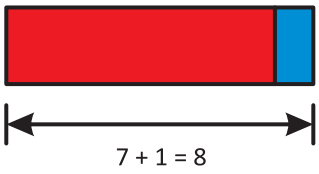
What are the lengths of the pieces?





Solution: 1, 3, 9, 27.

The trick with this one is realizing that "using the pieces in tandem" means using the sum *or difference* of two pieces to create measurements. For instance, if the ruler were cut into segments that measured 1, 7, 13, and 19 inches, the 1-inch and 7-inch segments could be placed end-to-end, and their sum would measure 8 inches, or the 13-inch and 19-inch segments could be placed side-by-side, and their difference would measure 6 inches.



With the set given above, it's possible to make the following measurements:

1, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 25, 26, 27, 31, 32, 33, 38, 39, 40.

Notice that the numbers occur in clusters. This happens because of the 1-inch segment. If the other pieces are used in tandem, the 1-inch piece can be used to make the number immediately above and below the result. Also notice that some efficiency is lost because the difference between consecutive pieces is always six: 19 – 13 = 6; 13 – 7 = 6; and 7 – 1 = 6.

These two observations can be helpful. The solution should include a 1-inch segment, and the segments should not have a common difference. From there, we can "build" a set intuitively.

Start with a 1-inch segment. If the next smallest segment is 2 inches, it will be possible to measure lengths of 2 and 3 inches, but if a 3-inch segment is included instead, then it's possible to get measurements of 2, 3 and 4 inches, which is obviously better. What if a 4-inch segment were used instead of the 3-inch segment? That would allow for measurements of 3, 4, and 5 inches, but there would be no way to get a measurement of 2 inches. There will be similar gap if a segment larger than 4 inches is used. Consequently, the second smallest segment should measure 3 inches.

With a 1-inch and 3-inch segment, measurements of 1, 2, 3, and 4 inches are possible, which means that measurements for 4 inches on either side of the two remaining numbers would be possible. A little thought reveals that a 9-inch segment would be a good choice; combined with the measurements from the 1-inch and 3-inch segments, any measurement from 5 to 13 inches would then be possible. Choosing a segment less than 9 inches would allow for duplication of some measurements, and choosing a segment greater than 9 inches would cause a gap.

Finally, the sum of the three segments chosen thus far is 1 + 3 + 9 = 13. Consequently, the remaining segment must be 40 – 13 = 27 inches. This works well, since any measurement from 1 to 13 inches is possible using the 1-, 3-, and 9-inch segments, and combining these measurements with 27 inches allows for any measurement from 27 – 13 = 14 up to 27 + 13 = 40 inches.