



This brainteaser was written by Derrick Niederman.

A *Friedman number* is a number that can be represented with an expression that uses only the digits in the number. In addition, the expression can include $+$, $-$, \times , \div , exponents and parentheses, but nothing else. For instance, 25 is a Friedman number because it can be represented as 5^2 . A *nice Friedman number* is a Friedman number for which the digits occur in the same order in the expression as they do in the original number. So, 343 is a nice Friedman number, because it can be represented by an expression with the digits 3, 4, and 3 in the same order:

$$343 = (3 + 4)^3$$

The first seven nice Friedman numbers are 127, 343, 736, 1285, 2187, 2502, 2592. Can you find an expression for each of them?



Solution: The expressions are shown below.

What's interesting to notice is that all the expressions require the use of at least one exponent, but this should not be surprising; we should expect that to convert a few digits to a large result, more than just the four binary operations might be needed.

One strategy for finding an expression is to determine the factors of the number, and then consider how those factors could be created from the digits. For instance, $2187 = 3^7$. Since the last digit of 2187 is a 7, then the question reduces to, is there some way to use 2, 1, and 8 to make 3? Indeed there is, as shown in the solution.

Another strategy is to consider how the addition or subtraction of one of the digits could lead to a value that is close to the original number. For example, the number 127 is one less than $128 = 2^7$, which leads directly to the solution shown below.

$$127 = -1 + 2^7$$

$$343 = (3 + 4)^3$$

$$736 = 7 + 3^6$$

$$1285 = (1 + 2^8) \times 5$$

$$2187 = (2 + 1^8)^7$$

$$2502 = 2 + 50^2$$

$$2592 = 2^5 \times 9^2$$