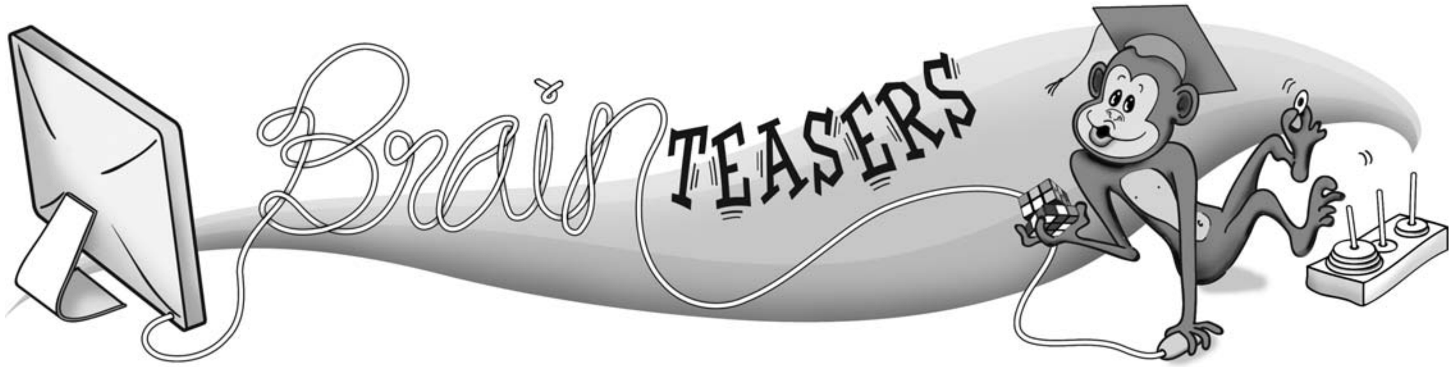

This brainteaser was written by Derrick Niederman.

Find the three-digit number *abc* equal to the value of the expression $(10 - a)(10 - b)(10 - c)$, where $a \neq 0$. That is,

$$abc = (10 - a)(10 - b)(10 - c),$$

where the left side of the equation is a three-digit number but the right side of the equation is the product of three numbers. (There is only one solution with $c \neq 0$. You are welcome to find the three additional solutions where $c = 0$, as well.)

Solution: The solution with $c \neq 0$ is 315.

Translating the problem to algebra gives 100*a* + 10*b* + *c* = (10 – *a*)(10 – *b*)(10 – *c*). This is an equation in three unknowns, which cannot be solved by conventional methods.

A possible strategy, then, is to choose a value for one of the digits and see what happens. A reasonable choice is to assume that *c* = 5, because 10 – *c* = 10 – 5 = 5 will then be one of the factors. Substituting *c* = 5 into the equation above gives the following:

$$\begin{aligned} 100a + 10b + c &= (10 - a)(10 - b)(10 - c) \\ 100a + 10b + 5 &= (10 - a)(10 - b)(5) \\ 20a + 2b + 1 &= 100 - 10a - 10b + ab \\ 30a - ab &= 99 - 12b \\ a &= \frac{99 - 12b}{30 - b} \end{aligned}$$

Though this looks like a mess, the result is actually very helpful. Testing possible values for *b* leads to the integer solution *a* = 3 when *b* = 1. (All other integer values of *b* yield noninteger values for *a*.)

When *a* = 3, *b* = 1, and *c* = 5, the result is 315 = (10 – 3)(10 – 1)(10 – 5) = 7 × 9 × 5.

The three solutions with *c* = 0 are 180, 350, and 500, and can be found with a similar approach.