Answer Key - Puzzling Pentominoes

As you investigate properties of pentominoes, refer to the names below to make it easier to share your results with others later.

1. Other than P, choose any two pentominoes. Combine them so they connect along a single square’s edge. Draw your combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

   The area of the combined figure is 10 in². The perimeter is 22 in.

2. Choose two pentominoes you didn’t choose in Question 1, again not using P. Combine them so they connect along a single square’s edge. Draw your new combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

   The area of the combined figure is 10 in². The perimeter is 22 in.
3. How do the areas and perimeters for the figures in Questions 1 and 2 compare? Can you think of an explanation for your observations? Do you think they would hold for any combination of two pentominoes? Explain your thinking mathematically.

The area of each figure is 10 in\(^2\). The areas are the same because each pentomino has an area of 5 in\(^2\), and when pentominoes are combined, areas are added together. The perimeter of both figures is 22 in. Each pentomino used has a perimeter of 12 in. By connecting a pentomino along one square’s edge, that edge becomes part of the interior, rather than part of the perimeter. Therefore, you have 

\[(12 - 1) + (12 - 1) = 22 \text{ in.} \]  

(Note: Students are not likely to find the perimeter pattern initially. It becomes more clear as they work through the activities.)

4. Combine pentomino P and another pentomino so they connect along a single square’s edge. Draw your combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

The area of the combined figure is 10 in\(^2\). The perimeter is 20 in.

5. Compare your results in Question 4 to your previous results. Can you think of a mathematical explanation for your observations?

The area is still the same and follows the same pattern. The perimeter in Question 4 is smaller than in previous questions. This is because the perimeter of pentomino P is less. It is 10 in, unlike all other pentominoes, which have perimeters of 12 in. It does still follow the pattern from Answer 3:

\[(10 - 1) + (12 - 1) = 20 \text{ in} \]

6. Using any number of pentominoes, create several combined figures and make observations about the areas of the figures. Write a rule to predict the area given the number of pentominoes.

Area = 5 \times (\text{number of pentominoes})

7. What is the least perimeter of a combined figure with two pentominoes connected along a single square’s edge? Explain how you know that you found the correct solution.

Any combination of pentomino P and another pentomino would have a perimeter of 20 in. There is no lesser perimeter because pentomino P has the least perimeter of all single pentominoes and all other pentominoes have equal perimeters.
8. What is the greatest perimeter of a combined figure with two pentominoes? Explain how you know that you found the correct solution.

The greatest perimeter would use two pentominoes with the largest perimeter (any pentominoes other than P) and connect them along a single’s square edge. This results in a perimeter of 22 in, the same result as in Questions 1 and 2.

9. For the given number of pentominoes, find a combined figure with the greatest perimeter.

<table>
<thead>
<tr>
<th>NUMBER OF PENTOMINOES</th>
<th>SKETCH</th>
<th>NAMES OF PENTOMINOES USED</th>
<th>PERIMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>[Answers vary.]</td>
<td>[Answers vary, but should not include pentomino P]</td>
<td>[22 in]</td>
</tr>
<tr>
<td>3</td>
<td>[Answers vary.]</td>
<td>[Answers vary, but should not include pentomino P]</td>
<td>[32 in]</td>
</tr>
<tr>
<td>4</td>
<td>[Answers vary.]</td>
<td>[Answers vary, but should not include pentomino P]</td>
<td>[42 in]</td>
</tr>
<tr>
<td>5</td>
<td>[Answers vary.]</td>
<td>[Answers vary, but should not include pentomino P]</td>
<td>[52 in]</td>
</tr>
</tbody>
</table>

10. Using the table in Question 9, predict the greatest perimeter of a combined figure with six pentominoes without creating it. Then, create it to check your result. Were you right? Write a rule to predict the greatest perimeter given the number of pentominoes.

With six pentominoes, the greatest perimeter is 62 in. The rule is:

\[
greatest \ perimeter = \left( (\text{previous perimeter}) - 1 \right) + (12 - 1)
\]

This can be simplified to:

\[
greatest \ perimeter = (\text{previous perimeter}) + 10
\]

This works with up to 11 pentomino because they all have a perimeter of 12 in.

11. Using all the pentominoes, create the combined figure with the greatest perimeter. Sketch your combined figure. Explain how you know that you found the correct solution.

The combined figure with the greatest perimeter will have each pentomino connected to any other pentomino along a single square’s edge without any connections being adjacent. The easiest way to build it is to make a “train.”