Answer Key – Building Polynomial Functions

1. What is the equation of the linear function shown to the right?

$y = (-2/3)x + 2$ or an equivalent form.

2. How did you find it?

Answers will vary. For example, students can use the slope-intercept form or the point-slope form to find the equation.

3. The slope – y-intercept form of a linear function is $y = mx + b$.
If you've written the equation in another form, rewrite your equation in slope – y-intercept form.

$y = (-2/3)x + 2$

4. Now, factor out the slope, and rewrite the function as $y = m\left(x + \frac{b}{m}\right)$.

$y = (-2/3)(x - 3)$

5. Choose a second linear function and write it in slope – y-intercept form.

Lines will vary.

6. Graph the function on the axis above, and be sure to label it.

Lines will vary.

7. Rewrite your second function with the slope factored out (just like you did in Question 4).

Lines will vary.

8. For each function, what does $\frac{b}{m}$ represent on the graph?

$-\frac{b}{m}$ is the x-intercept.

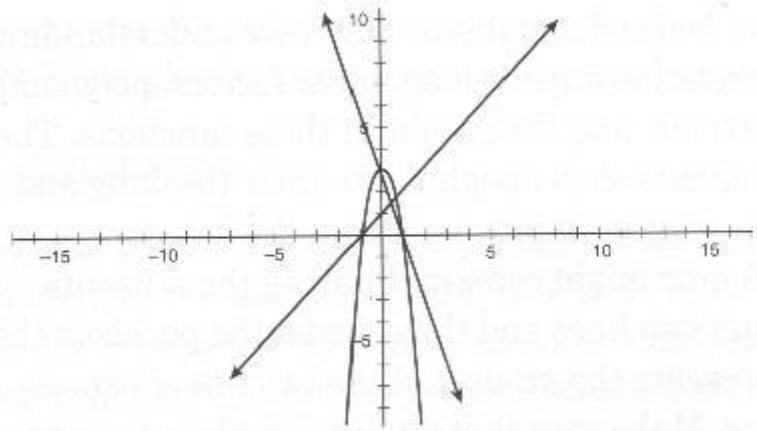
If you let $c = -\frac{b}{m}$, then the form $y = m(x - c)$ could be called the slope – x-intercept form of a linear equation, where c is the x-intercept. The factor theorem states that if c is a root (x-intercept) of a polynomial function, then $(x - c)$ must be a factor of that polynomial function. Note that $(x - c)$ is a factor of the expression. The only other factor is the slope m.

9. From their slope – y-intercept form, multiply the two functions together.

Answers will vary.

10. Graph the resulting function on the same axis as the two lines on the previous page.

Answers will vary. A sample graph is given below.



11. What kind of function did you get?

A quadratic function.

12. What relationship do you see between the graph from Question 10 and the lines?

The product of the two linear expressions gives the parabolic expression. Students may not immediately see this.

- …and the x-intercepts?

The lines have the same x-intercepts as the parabola.

- …and the *y*-intercepts?

The y-intercept of the parabola is the product of the y-intercepts of the lines.

13. Identify the left-most *x*-intercept on the graph. With a straight-edge, cover everything to the right of that point. What connections do you see relating the signs of the *y*-values?

Answers will vary. The teacher should assist students who are placing their paper strip incorrectly.

14. Identify the right-most intercept on the graph. With a straight-edge, cover up everything to the left of that point. What connections do you see relating to the signs of the *y*-values?

Answers will vary. The teacher should assist students who are placing their paper strip incorrectly.

Complete the following sentences.

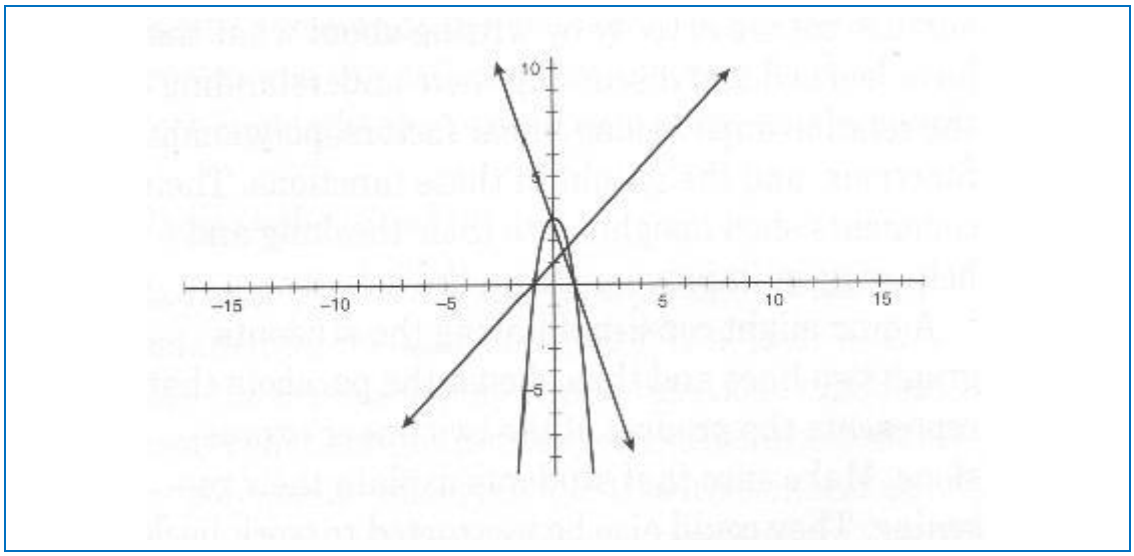
15. When both lines are **above** the *x*-axis, the *y*-values are positive and the parabola is above the x-axis.

16. When both lines are **below** the *x*-axis, the *y*-values are negative and the parabola is above the x-axis.

17. When one line is above the *x*-axis and the other is below the *x*-axis, the parabola is below the *x*-axis.

y-VALUE OF l_1	*y*-VALUE OF l_2	PARABOLA IS ABOVE/BELOW THE *x*-AXIS
+	+	above
+	-	below
-	+	below
-	-	above

18. Based on the patterns you saw on the previous page, draw a sketch of the quadratic function that would be obtained from the linear expressions of these lines.



19. Write the equation for each line.

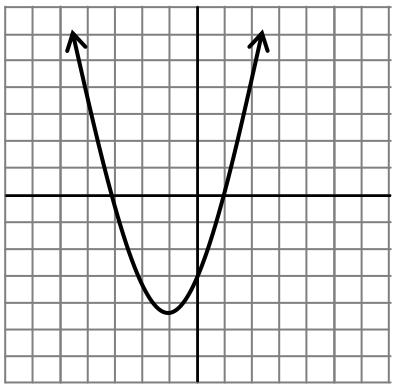
$y = -3x + 3$ and $y = x + 1$

20. To check your sketch in Question 18, multiply the expressions together, and graph the resulting function on the grid above.

Quadratic function: $y = -3x^2 + 3$

Answer Key – Working Backwards

To the right, a parabola is given. Working backwards, we want to find two lines that could represent the linear factors.



1. What is the left-most x-intercept of the parabola?

(-3,0)

2. To the left of that point, what do we know about the two lines that represent the linear factors?

Answers may vary. A possible response could be that the two lines must both have positive y-values or negative y-values in order for the parabola to have a positive value.

3. What is the right-most x-intercept of the parabola?

(1,0)

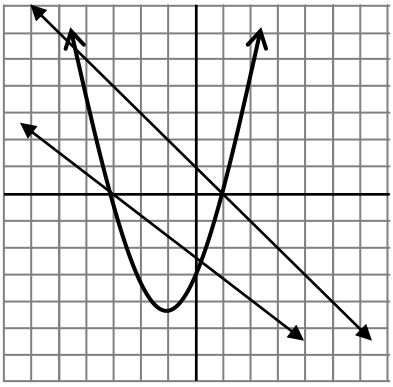
4. To the right of that point, what do we know about the two lines that represent the linear factors?

Answers may vary. A possible response is that the two lines must both have positive y-values or negative y-values in order for the parabola to have a positive value.

5. What do we know about the two lines between the two linear factors?

Answers may vary. A possible response is that if one line has positive y-values, the other must have negative y-values (and vice versa).

6. On graph sketch two lines that could represent the linear factors.



Lines will vary.

7. Write the equations for the lines you sketched.

Equations will vary.

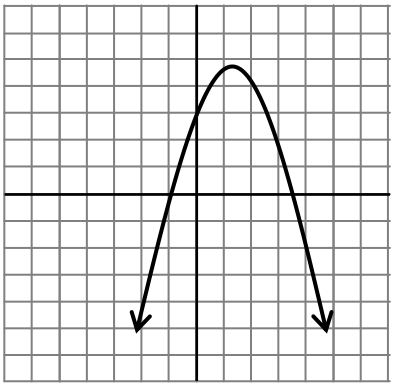
8. Multipty the two expressions together.

Answers will vary.

9. Graph the resulting parabola. How does it compare to the graph given?

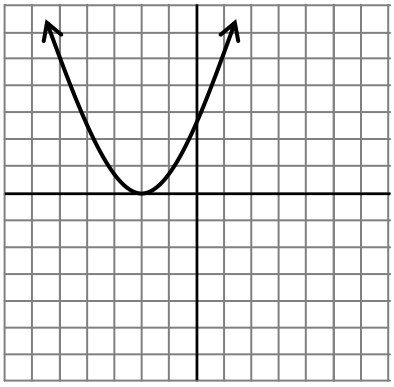
Graphs will vary; however, the graphs should be close.

10. Follow the same steps for the parabola to the right to determine a possible equation. Below, describe how you got the equation you've found.



Equations will vary. One possible equation is: $y= -x^2+2.4x+3.36$. The product of the *y*-intercepts of the lines should equal the *y*-intercept of the parabola. The parabola should have negative *y*-coordinates when just one of the lines has negative *y*-coordinates.

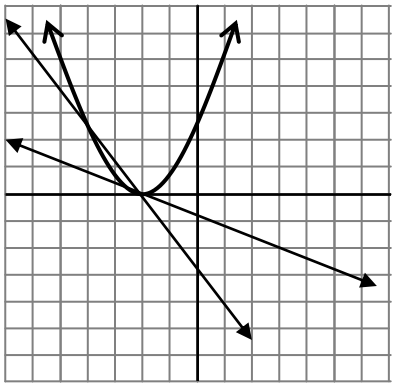
11. What is different about the x-intercept(s) of the graph to the right?



There is only one at (-2,0).

12. Sketch the lines that could represent the factors.

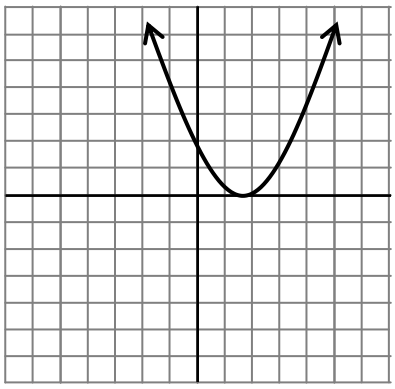
Sketches will vary



13. Describe the process of how you determined your answer for Question 12.

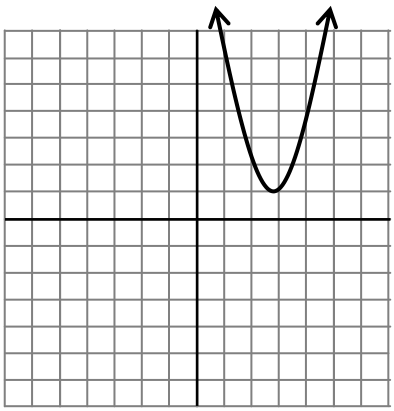
Answers will vary. Students should be aware that both lines need to have the same x-intercept at (-2,0). Furthermore, students should be checking the y-values of both linear functions to make sure that the product always stays positive. The product of the *y*-intercepts of the lines should also equal the *y*-intercept of the parabola.

14. To the right is another graph just like the one above. Find an alternative pair of lines that could also satisfy the requirements for representing the factors. Graph them over the parabola to the right.



Lines and graphs will vary. If you reflect the two lines in #12 across the line x = -2, you will get an alternative pair of lines.

15. For the graph to the right, sketch the lines that could represent the factors.



No lines can be drawn that would be components of the quadratic function given.

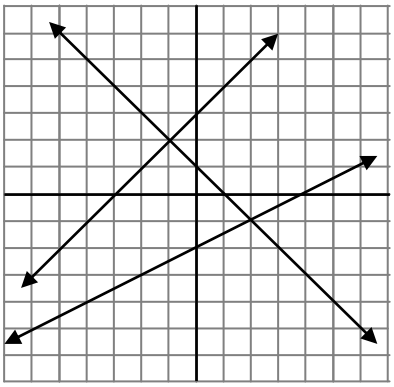
16. Describe the process of how you determined your answer for Question 15.

The absence of x-intercepts implies that no real roots exist. This is, lines cannot be drawn, because the quadratic equation cannot be factored into linear expressions over the real numbers.

Answer Key – Building Polynomial Functions of Degree 3

1. Graph each of the following three linear functions on the axis provided.

$$y_1 = \frac{1}{2}x - 2$$
$$y_2 = x + 3$$
$$y_3 = -x + 1$$



2. Multiply the three functions together.

$y = -(1/2)x^3 + x^2 + (11/2)x - 6$

3. What type of function is the result?

A cubic function.

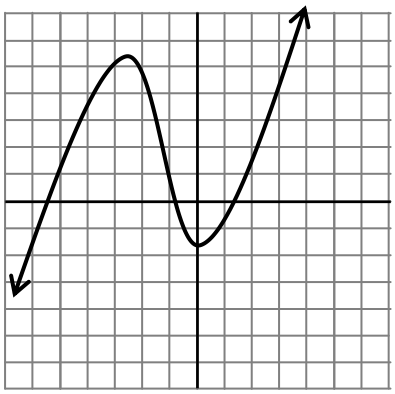
4. Using what you know about multiplying signed numbers and the graphs of the lines, sketch your prediction for the graph of the product of the three linear functions.

Graph should have a y-intercept of (0,-6); the x-intercepts are located at (-3,0), (1,0), and (4,0). The local maximum is (2.69,6.3), while the local minimum is (-1.36,-10.37).

5. Describe in your own words how you chose your graph.

The graph drawn should go through the *x*-intercepts of all three lines. The product of the *y*-intercepts of the lines gives the *y*-intercept of the cubic. Help in graphing the cubic can also be obtained by observing the signs of the *y*-coordinates.

6. Work-backwards: Find three lines that could be the components for the cubic represented by the graph to the right.



7. Describe in your own words how you chose your lines.

6-7. Each line should pass through one of the *x*-intercepts. The product of the *y*-intercepts of the lines should be around -1.5. Exactly one or exactly three of the lines will have negative *y*-coordinates when the *y*-coordinates on the cubic are negative.