

Answers to Embedded Figures Activity Sheets

Sheet 1:

1. (a) 4

(b) 1

(c) 5

2. (a) 9

(b) 4

(c) 1

(d) 14

3. (a) 16

(b) 9

(c) 4

(d) 1

(e) 30

4. $25 + 16 + 9 + 4 + 1 = 55$.

5. $64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 204$.

In general, for an $n \times n$ square, $1^2 + 2^2 + \dots + n^2$ squares are possible. It can be shown that the sum is equal to $n(n+1)(2n+1)/6$.

Sheet 2:

1. (a) 6

(b) 4

(c) 2

(d) 3

(e) 2

(f) 1

(g) total is 18.

2(a) 12 9 6 3

 8 6 4 2

 4 3 2 1

Several patterns can be seen in this array. The bottom row contains consecutive integers from 1 to 4. The middle row is the bottom row multiplied by 2. The top row is the bottom row multiplied by 3. A similar analysis may be made on the basis of columns.

(b) 60

3. The answer will look like the array in 2(A).

Some students will notice that 15 and 21 are *triangular numbers*. In general, the n th triangular number is the sum of the first n positive integers.

$T_n = 1 + 2 + \dots + n = n(n+1)/2$ (see Phillips et al. 1991). A generalization for this problem is that the total number of rectangles in an m row \times n column rectangle is $T_m \times T_n = mn(m+1)(n+1)/4$.

(table on next page)

No. of rows	No. of Columns					Total in Row
	1	2	3	4	5	
1	30	24	18	12	6	$90 = 6 \times 15$
2	25	20	15	10	5	$75 = 5 \times 15$
3	20	16	12	8	4	$60 = 4 \times 15$
4	15	12	9	6	3	$45 = 3 \times 15$
5	10	8	6	4	2	$30 = 2 \times 15$
6	5	4	3	2	1	$15 = 1 \times 15$

Grand total = $(1 + 2 + 3 + 4 + 5 + 6) \times 15 = 21 \times 15 = 315$

4. In view of the previous discussion, the total number will be $(T_4)^2 = 10^2 = 100$. Many students will arrive at this result by making a table similar to the foregoing ones. Be sure that your students recognize that squares are rectangles.

Sheet 3:

1. 5 (4 small and 1 large).

2. 13 (9 small, 3 medium, and 1 large).

3. Answers will vary. Some students may predict 25 on the following basis:

Size of Triangle	No. of Triangles	Difference
1	1	
2	5	4
3	13	8
4	25?	12?

If this sequence were the actual pattern, the number of triangles would be a quadratic function of the size of the large triangle. In fact, the relation is more complex.

4. (a)	3	1		(b)	6	3	1	
	1	0			3	0	0	
	4	1	Grand total = 5		9	3	1	Grand total = 13
(c)	10	6	3	1				
	6	1	0	0				
	16	7	3	1	Grand total = 27			

	Size 1	Size 2	Size 3	Size 4	Size 5	
Up	15	10	6	3	1	
Down	10	3	0	0	0	
Total	25	13	6	3	1	Grand total = 48

6. All are triangular numbers. For the up triangles the numbers will be 21, 15, 10, 6, 3, and 1. The down triangles will be 15 of size 1, 6 of size 2, and 1 of size 3. The pattern is found by counting down every other triangular number starting with the number found in the second column of the first row.

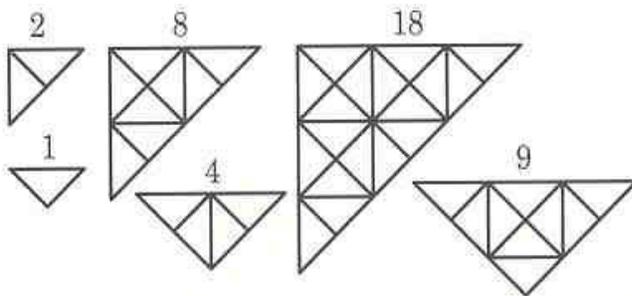
7. Down triangles can never be more than half the size of the large triangle. So the largest down triangle in a triangle measuring 8 units will be of size 4. In a triangle measuring 11 units it will be of size 5.

8. For the up triangles, find the first 10 triangular numbers. Listed backward, as they would appear in the table, they are 55, 45, 36, 28, 21, 15, 10, 6, 3, and 1. The down triangles follow the pattern of every other triangular number starting with 45:45, 28, 15, 6, and 1. Check this result by observing that exactly 1 down triangle is of size 5. It is formed by joining the midpoints of the sides of the large triangle of size 10.

Finding the Number of Side Triangles is Trickier

Sheet 4: Four triangles can be found of each size, for a total of 8. 2. Let the "size" of a triangle be determined by how many of the smallest triangles are contained in it. Then make a chart.

Size of Triangle	No. of Triangles
1	16
2	16
4	8
8	4
Total = 44	



3.

<u>Size of Triangle</u>	<u>No. of Triangles</u>	
1	36	
2	36	
4	24	
8	16	
9	8	
18	4	Total = 124

<u>4. Size of Triangle</u>	<u>No. of Triangles</u>	
1	64	
2	64	
4	48	
8	36	
9	24	
18	16	
16	12	
32	4	Total = 268

5. In the $n \times n$ square are *side triangles* of size $1^2, 2^2, \dots, n^2$, which are formed by the side of the square and parts of two diagonals. In addition, corner triangles having size $2(1^2), 2(2^2), \dots, 2n^2$, are formed by two sides and the diagonal of a square.

It is easier to count the corner triangles. Each of the $(n^2 + (n-1)^2 + \dots + 2^2 + 1^2)$ squared has 4 corner triangles, one in each corner. So the total number of corner triangles is $4(n^2 + (n-1)^2 + \dots + 2^2 + 1^2)$.

Finding the number of side triangles is trickier. In a large square of side n , the number of side triangles with hypotenuse m , that is, those of "size" m^2 , is given by $4(n+1-m) \text{ int}(n+1-m/2)$, where "int" is the greatest integer function. Some students may be able to discover this relationship as a recursive pattern without expressing it in this form.