

How to Solve It!



ILLUMINATIONS
Resources for Teaching Math

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How to Solve It

George Polya's Four-Step Method

1. Understand the Problem
2. Devise a Plan
3. Carry Out the Plan
4. Look Back

Polya, G. (1957). *How to solve it: A new aspect of mathematical method*, 2nd ed. Princeton, NJ: Princeton University Press.

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Polya Illustrated

[Find the Number
\(logic problem\)](#)

[Help Wanted
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Find the Number

Use the ten clues, in order, to find the mystery number.

1. It is a five-digit whole number.
2. It is divisible by 5.
3. It is divisible by 4.
4. The sum of its ten-thousands digit and thousands digit is 14.
5. The sum of its ten-thousands digit and its hundreds digit is 11.
6. The sum of its thousands digit and its tens digit is 8.
7. The sum of its hundreds digit and its units digit is 3.
8. The sum of its tens digit and its units digit is 2.
9. It is greater than 80,000.
10. Its thousands digit is 6.

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1. Understand the Problem

- Read the problem carefully. You'll have to focus on using the clues IN ORDER.
- Be sure you understand the terms
 - What is a “five-digit whole number”?
 - What does “divisible” mean?
 - What are units digits, tens digit, etc.?
- Yes, these may seem very easy... but that may not always be the case.

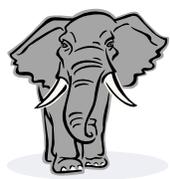
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1. Understand the Problem

- ✓ Break it into parts.
- ✓ Do you have all the tools you need?
 - ✓ Enough information
 - ✓ Appropriate skills
- ✓ Can you draw a picture?
 - ✓ Artistic skill not required.

Q: How do you eat an elephant?



A: One bite at a time!

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1. Understand the Problem

- Introduce notation and variables.
- Clue (1) tells us we have a 5-digit number. Without knowing anything else, we can write it as follows:

$$ab,cde$$



2. Devise a Plan

- Follow the clues systematically
- Use rules of divisibility to limit possibilities
- Use algebraic equations to find digits
- If the problem is too hard, try doing an easier problem
 - In this problem, you could rearrange the clues. BUT...when you look back you'll realize that doesn't quite answer the question.



3. Carry Out the Plan

- From Clue (1):
 - Form of the number is ab,cde .
- From Clue (2):
 - Divisible by 5 — therefore, the units digit is 0 or 5.
- From Clue (3):
 - Divisible by 4 — therefore, it is even, and the last two digits must be divisible by 4. So, the units digit is 0, and the tens digit is 2, 4, 6, or 8.



3. Carry Out the Plan

- | | |
|--|------------------------------|
| (1) ab,cde | If $e = 0$ and $c + e = 3$, |
| (2, 3) d is even,
and $e = 0$ | then $c = 3$ |
| (4) $a + b = 14$ | So, $a = 8$ |
| (5) $a + c = 11$ | Then, $b = 6$ |
| (6) $b + d = 8$ | And, $d = 2$ |
| (7) $c + e = 3$ | The number is... |
| "Aha! I have enough info.
I don't need any more clues." | 86,320 |



4. Look Back

- Is it reasonable? Does it satisfy the clues... including the clues we didn't need?
 - ✓ (8) $d + e = 2$
 - ✓ (9) greater than 80,000
 - ✓ (10) thousands digit is 6
- We used 7 clues.
 - Could we have solved the problem with fewer?
- Did we answer the question? Don't just give a , b , c , d , and e . The problem asks for a five-digit number.



4. Look Back

- What did I learn from this problem?
 - Review simple algebra skills
 - Organized, logical thinking
- How can I use it to solve other problems?
- Could I have used another method?
 - Trial and error, maybe (but why???)

[Find the Number
\(logic problem\)](#)

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Help Wanted

Serge works at the supermarket as a cashier, a bagger, and a stocker. He earns \$7.00 per hour as a cashier, \$6.00 per hour as a bagger, and \$5.00 per hour as a stocker. In a given week, Serge works 4 hours as a cashier, 9 hours as a bagger, and 7 hours as a stocker. What is Serge's average pay per hour?

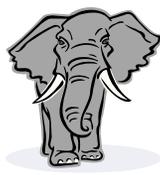
1. Understand the Problem

- Read it carefully. Note that you're asked for his average pay per hour.
- Be sure you understand the terms.
 - Does it matter what jobs he has, or only that he has three jobs?
- Yes, this may seem very easy... but that may not always be the case.

1. Understand the Problem

- ✓ Break it into parts.
- ✓ Do you have all the tools you need?
 - ✓ Enough information?
 - ✓ Appropriate skills?
- ✓ Can you draw a picture?
 - ✓ Artistic skill not required.
 - ✓ A chart is a picture.

Q: How do you eat an elephant?



A: One bite at a time!

1. Understanding the Problem

- What information do we have? How can we organize it?

Cashier	\$7.00 per hour	4 hours
Bagger	\$6.00 per hour	9 hours
Stocker	\$5.00 per hour	7 hours

2. Devise a Plan

- Consider one job at a time.
- Find earnings for each job.
- Find total earnings and total hours.
- Find the average.

3. Carry Out the Plan

- Jobs
 - Cashier: $4 \text{ hr} \times \$7/\text{hr} = \28
 - Bagger: $9 \text{ hr} \times \$6/\text{hr} = \54
 - Stocker: $7 \text{ hr} \times \$5/\text{hr} = \35
- Total pay: $\$28 + \$54 + \$35 = \117
- Total hours: $4 + 9 + 7 = 20 \text{ hr}$
- Average pay: $\$117 \div 20 \text{ hr} = \$5.85/\text{hr}$

4. Look Back

- Is it reasonable?
- What did I learn from this problem?
 - The answer is **not** \$6.00
- How can I use it to solve other problems?
- How else could I have solved the problem?
 - Weighted average

Find the Number
(logic problem)

Help Wanted
(algebra problem)

Process Review

Student Reflection

Process Review

George Polya's Four-Step Method

1. Understand the Problem
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4. Look Back

1. Understand the Problem

- READ the problem.
- What are you expected to find: unknowns
- What are you given: data, situation or condition
 - Do you have enough information?
 - Is any information unnecessary or contradictory?
- Explain the problem in your own words
 - Break it into parts
- Draw a picture or diagram
- Introduce suitable notation and variables

2. Devise a Plan

- Look at the unknown.
 - Do you know how to solve problems with this type of unknown?
- Can you find a connection between the data and the unknown?
- Have you seen a problem like this before?
- Do you know any similar problems you can use to help you?

2. Devise a Plan

If it's too hard...

- READ it again!
- Do you understand the problem? Redo Step 1.
- Can you restate the problem?
- Can you break the problem into pieces?
- If you cannot solve the proposed problem...
 - Try to first solve a related (maybe easier) problem
- Did you use all the data? ...the whole condition?

3. Carry Out the Plan

- Follow your plan
- CHECK EACH STEP
- Can you see clearly that each step is correct?
- If it doesn't lead to a solution...
 - Return to Step 2. Pick another plan!

4. Look Back

- Can you check your result?
- Can you check your logic?
- Did you answer the question?
- Is it reasonable?
- Can you do the problem differently?
- Can you use your results to help you solve other problems?
- Have you learned something from this problem?

Student Reflection

Think about it...

- Which step do you think is most difficult?
- Which step do you think is most important?
- How can Polya's method help you with
 - other types of math problems?
 - problems from other areas of school and life?
- How can metacognition help you with problem solving?

Try It Yourself

Use the ten clues, in order, to find the mystery number.

1. It is a four-digit whole number.
2. It is greater than 4,000.
3. The sum of its hundreds digit and its units digit is 9.
4. Twice its tens digit is 2 more than its thousands digit.
5. The sum of one-fifth of its hundreds digit and two-thirds of its units digit is 6.
6. Its tens digit is one less than its thousands digit.
7. The product of its hundreds digit and its ones digit is 0.
8. It is not an even number.
9. It is less than 5,000.
10. Its tens digit is 3.

Try It Yourself

- Use a full sheet of paper for your problem.
- Identify each step of the process.
 - Allow space for each step of the process.
- A book club promises to send 8 books for \$1, if you join the club. After you receive the first 8 books, you may select more books at a rate of \$19.99 per book. If you spend a total of \$80.96, how many extra books did you purchase?