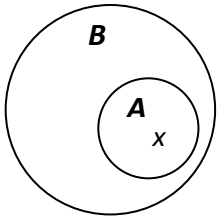
Reference Sheet

The following symbols are used on this sheet:

$\subset$	$\in$	$\notin$	$\cap$	$\therefore$
"is a subset of"	"is an element of"	"is not an element of"	"intersects"	"therefore"

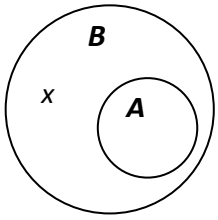
Direct Reasoning



THE UNDERLYING IDEA

If $A \subset B$ and $x \in A$, then we can conclude that $x \in B$.

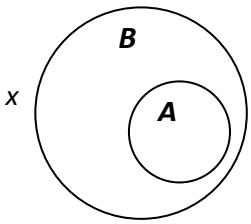
Premise:	All elements in A are also in B.	$p \rightarrow q$:	If an element is in A, then it is in B.
Premise:	x is an element in A.	p:	x is an element in A.
Conclusion:	x is an element in B.	$\therefore$ q:	x is in B.



The argument is VALID because if A is in B and x is in A, then we can be 100 percent certain that x is in B.

But watch out! If $A \subset B$ and $x \in B$, we cannot be 100 percent certain that $x \in A$. The argument would be invalid. The diagram to the right shows that $A \subset B$ and $x \in B$, but $x \notin A$.

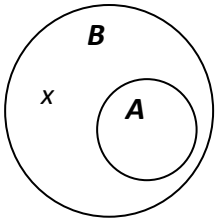
Indirect Reasoning



THE UNDERLYING IDEA

If $A \subset B$ and $x \in B$, then we can conclude that $x \in A$.

Premise:	All elements in A are also in B.	$p \rightarrow q$:	If an element is not in B, then it is not in A.
Premise:	x is not an element in B.	p:	x is not an element in B.
Conclusion:	x is not an element in A.	$\therefore$ q:	x is not in A.

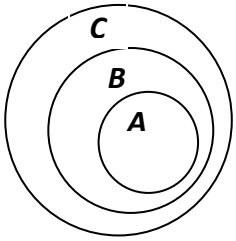


The argument is VALID because if A is in B and x is not in B, then we can be 100 percent certain that x is not in A.

But watch out! If $A \subset B$ and $x \notin A$, we cannot be 100 percent certain that $x \notin B$. The argument would be invalid. The diagram to the right shows that $A \subset B$ and $x \in B$, but $x \notin A$.

ILLUMINATIONS Resources for Teaching Math
© 2013 National Council of Teachers of Mathematics
http://illuminations.nctm.org

Transitive Reasoning



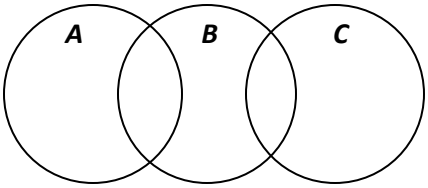
THE UNDERLYING IDEA

If $A \subset B$ and $B \subset C$, then we can conclude that $A \subset C$.

Premise:	All elements in *A* are also in *B.*	$p \rightarrow q$:	If an element is in *A*, then it is in *B*.
Premise:	All elements in *B* are also in *C*.	$q \rightarrow r$:	If an element is in *B*, then it is in *C*.
Conclusion:	All elements in *A* are also in *C*.	$\therefore$ $p \rightarrow r$:	If an element is in *A*, then it is in *C*.

The argument is VALID because if *A* is in *B* and B is in C, then we can be 100 percent certain that *A* is in *C*.

But watch out! The diagram below should that if $A \cap B$ and $B \cap C$, it is not necessarily true that $A \cap C$.



ILLUMINATIONS Resources for Teaching Math
© 2013 National Council of Teachers of Mathematics
http://illuminations.nctm.org