# Answer Key – Computer Animation

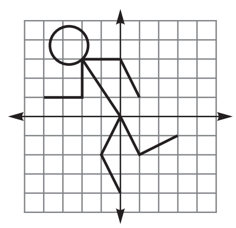
**1.** The first step in computer animation is for the animator to create a simplified representation of a character's anatomy, analogous to a skeleton. Enter key points in your image in a 2×*n* matrix, where the *x*-coordinate of a point is in the top row, and the *y*-coordinate of the point is in the bottom row. There will be one column for each point in your shape. Call this matrix *S* (for "skeleton").

$$S = \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$$

**2.** Find each product below, and graph the shape that results on a sheet of graph paper. Below each matrix, write a brief description of how the shape is "transformed" by the matrix multiplication. Use geometric terms such as reflection, rotation, stretching, translation. Be as precise as you can, telling where the line of reflection is, or in which direction the rotation occurs.

**a)** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} S =$

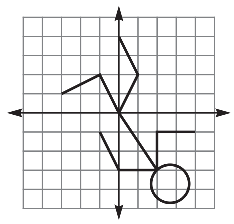
$$\begin{bmatrix} 1 & 0 & -2 & -2 & -4 & 0 & 1 & 3 & -1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \end{bmatrix}$$



Geometric Description: Reflection over the *y*-axis

**b)** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} S =$

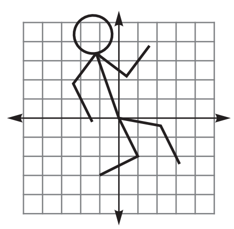
$$\begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ -1 & -3 & -3 & -1 & -1 & 0 & 2 & 1 & 2 & 4 \end{bmatrix}$$



Geometric Description: Reflection over the *x*-axis

**c)** $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} S =$

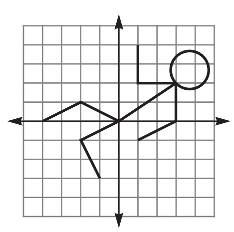
$$\begin{bmatrix} -\frac{7}{5} & -\frac{12}{5} & -\frac{6}{5} & \frac{2}{5} & \frac{8}{5} & 0 & 1 & -1 & \frac{11}{5} & \frac{16}{5} \\ -\frac{1}{5} & \frac{9}{5} & \frac{17}{5} & \frac{11}{5} & \frac{19}{5} & 0 & -2 & -3 & -\frac{2}{5} & -\frac{12}{5} \end{bmatrix}$$



Geometric Description: Counter-clockwise rotation through 53º

**d)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} S =$

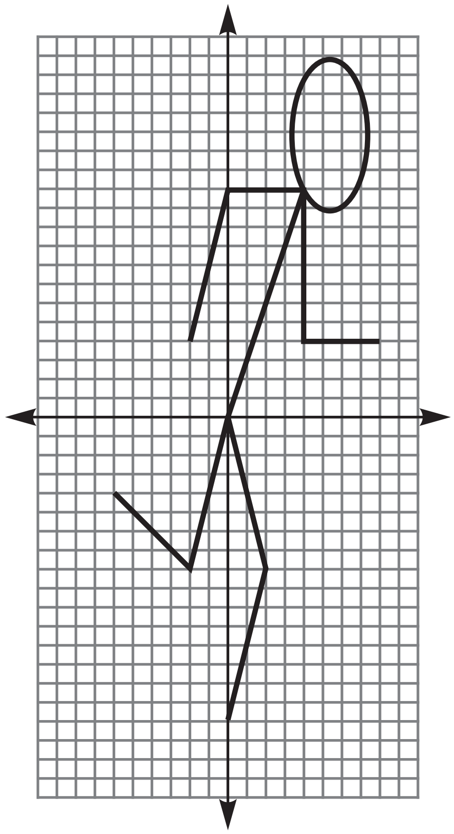
$$\begin{bmatrix} 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \\ -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \end{bmatrix}$$



Geometric Description: Reflection over the line $y = x$

**e)** $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} S =$

$$\begin{bmatrix} -2 & 0 & 4 & 4 & 8 & 0 & -2 & -6 & 2 & 0 \\ 4 & 12 & 12 & 4 & 4 & 0 & -8 & -4 & -8 & -16 \end{bmatrix}$$



Geometric Description: Stretch out twice as wide and four times as tall

**3.** The geometric descriptions you wrote for the transformations in Questions 2a and 2b are fairly similar. Explain why this makes sense, based on the matrices used. (**Hint:** Think about the process you used to find the matrix product.)

In the process of matrix multiplication, the matrix for Question 2a changes the signs of the *x*-coordinates, while the matrix for Question 2b changes the signs of the *y*-coordinates.

**4.** Find a 2×2 transformation matrix that will make your image shrink into a single dot. Do you think there is more than one matrix that would have this effect? Can you find a matrix that will flatten your image into a horizontal (or vertical) line?

The only matrix that will shrink the image to a single dot is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

The matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ will flatten the image into horizontal and vertical lines, respectively.

**5.** Find a 2×2 transformation matrix that will leave your image unchanged. Do you think there is more than one matrix that would have this effect?

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the only such 2×2 matrix. You may want to point out to students that this matrix has a special name; it is the identity matrix.

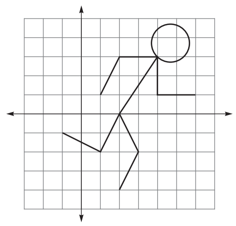
**6.** Represent your shape using a 3×*n* matrix adding a third row of all 1's to matrix *S* from Question 1. This extra row will allow for additional transformations, which you will explore below. Call this matrix *R*.

$$R = \begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

**7.** Compute each matrix product below, and again graph the resulting shapes. Use the row of 1's for your calculation, but ignore it when you graph. Write out the products, and then write a sentence describing how the image has been transformed.

**a)** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R =$

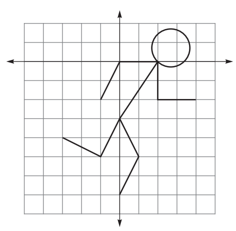
$$\begin{bmatrix} 1 & 2 & 4 & 4 & 6 & 2 & 1 & -1 & 3 & 2 \\ 1 & 3 & 3 & 1 & 1 & 0 & -2 & -1 & -2 & -4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



Geometric Description: Translation 2 units to the right

**b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} R =$

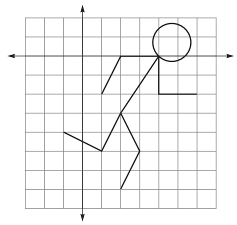
$$\begin{bmatrix} -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ -2 & 0 & 0 & -2 & -2 & -3 & -5 & -4 & -5 & -7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



Geometric Description: Translation 3 units down

**c)** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} R =$

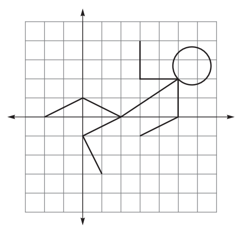
$$\begin{bmatrix} 1 & 2 & 4 & 4 & 6 & 2 & 1 & -1 & 3 & 2 \\ -2 & 0 & 0 & -2 & -2 & -3 & -5 & -4 & -5 & -7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



Geometric Description: Translation 2 units to the right and 3 units down

**d)** $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} R =$

$$\begin{bmatrix} 3 & 5 & 5 & 3 & 3 & 2 & 0 & 1 & 0 & -2 \\ -1 & 0 & 2 & 2 & 4 & 0 & -1 & -3 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



Geometric Description: Reflection over the line $y = x$ followed by a translation 2 units to the right

or

Translation 2 units up followed by a reflection over the line $y = x$